Indian Institute of Science

DS284 - Numerical Linear Algebra

Exercise 1

August 10, 2018

1. A is matrix of dimension $m \times n$ and B is of dimension $n \times p$ matrix.

   Let $C(.)$ and $R(.)$ denote the columns space and row space respectively. Given $D = AB$, Which of the following must be true and why? (More than one can be true)

   a) $C(D) \subseteq C(A)$

   b) $C(D) \subseteq C(B)$

   c) $R(D) \subseteq R(A)$

   d) $R(D) \subseteq C(B)$

   Now can you relate the rank of D to the rank of A and B?

2. The induced matrix norm is given by $||A||_{(m,n)} = \sup ||Ax||_{(m)}$, where x is a unit vector in $\mathbb{R}^n$ and $||.||$ corresponds to the p norm (p is a natural number). Using MATLAB/Octave create a matrix using:

   A = randn(80,100);

   Create random unit vectors x using:

   temp = randn(100,1);

   x = temp/norm(temp);

   check for multiple values of x (use a loop, check for about 1000 values) what is the maximum value of p norm for the vector $A * x$.?

   norm_of_Ax = norm(A*x, p);

Calculate the p norm of A using:

norm_of_A = norm(A, p);

Verify the equality $||A||_{m,n} = \sup ||Ax||_m$ for multiple values of p.

3. Show that:
   a) The eigen values of a skew-symmetric matrix must be complex.
   b) The eigen values of a symmetric matrix must be all real.
   c) Given any skew-symmetric matrix S, the matrix (I-S) must always be non-singular.

   (Create a random matrix using randn function in MATLAB. Create symmetric and skew symmetric matrices using $(A + A^T)$ and $(A - A^T)$ respectively. Use eig() function in MATLAB to verify the above).

4. Given a vector $u \in \mathbb{R}^n$, what can you say about the eigen values of $A = uu^T$ (u^T is the transpose of u). notice that A is a $n \times n$ matrix.

   What is the rank of A? (Use what you learnt in the first question) Use the rank() and eig() functions in MATLAB to verify.