Indian Institute of Science

DS284 - Numerical Linear Algebra

Exercise 2

September 12, 2018

1. Implement your own routines for Classical and Modified Gram-Schmidt. Use, the code at `https://github.com/dugarab/ds284/tree/master/gram-schmidt`.

   In the **gram_schmidt_main.m** file, replace the default function calls with your function call and then check correctness. (In Essence the diagonal values of your **R** matrix should map similar to the diagonal matrix in the plot).

   Realize how in the **gram_schmidt.m** function provided only changing one variable changes Classical Gram Schmidt to Modified Gram Schmidt. Analyse.

2. Given below is a magic matrix of size 3(What's so magical about it?) :

$$\begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$$

   Find the Q,R factorization of the given matrix by hand.

   Now do the following on MATLAB/Octave:

   [Q,R] = qr(magic(3));

   Do these Q and R match your Q and R? Is the QR factorization unique? If not unique, can you impose a condition on R to make the factorization unique?

3. Geometrically, the orthogonal/unitary matrix is a matrix transformaion that preserves the 2-norm of a matrix and causes only rotation.

   Can you justify why (I - 2P) is a unitary matrix?

   Prove the same algebrically too (A matrix 'Q' is unitary if and only if $Q^TQ = I$).

4. Given a matrix A and its projection matrix P (P projects all vectors on to the columns space of A) , answer the folowing:
   a) If A is full rank, what is P?
   b) Given P is there an easy way to find the null space of A?
   c) What can you say about the eigen values of P? (hint: divide the whole $\mathbb{R}^n$ space into the column and null space of A)