An analysis of the limitations of university rankings and its use

A project report submitted in partial fulfilment
of the requirements for the degree of

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by

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Declaration of Originality

I, **Lubhawan Parihar**, with SR No. **06-18-01-10-51-21-1-19239** hereby declare that the material presented in the thesis titled

An analysis of the limitations of university rankings and its use

represents original work carried out by me in the **Department of Computational and Data Sciences** at **Indian Institute of Science** during the years **2022-2023**.
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In my capacity as supervisor of the above-mentioned work, I certify that the above statements are true to the best of my knowledge, and I have carried out due diligence to ensure the originality of the report.

Advisor Name: Prof. Murugesan Venkatapathi

Advisor Signature

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Abstract

In the first part of this work, we studied the university ranking systems and their pitfalls. Using the correlations between different ranking systems, and correlations within a ranking system for an offset of 2 yrs, it was highlighted that the ranks $\gtrsim 100$ are unreliable for QS and THE rankings. A similar result is also highlighted for NIRF (National Institute Ranking Framework - India) rankings, where numbers greater than 30 are unreliable. This serious drawback may also be a symptom of the decoupling of the indicators used and the actual parameters perceived to be measured, i.e. faculty-student ratio and the teaching quality, for example. Next, the dominance of the two most common indicators in most rankings, i.e. citations per faculty and academic reputation, was shown. With the correlation between these two indicators, the geographical bias in a well-known university ranking (QS) was demonstrated using Chi-square tests. Sensitivity tests on these rankings were also performed to show that a 25% noise in the score represents a 10% change in rankings.

In the work's second part, we model the graduate admission process in US universities, where the above rankings play a crucial role. We model the competitive non-cooperative application process (where an applicant is unaware of the distribution of test scores, academic performance, and the choices of applicants) to determine the effects of conflict among applications to different programs on the fairness of the admission process. Each applicant can apply to a small set of graduate programs in various universities, which amounts to a considerable sum of money every year. This process was modelled using a Sequential Monte Carlo approach with normal, uniform and exponential distribution of merit scores, where the final distribution of the admitted and declined students after the randomized process was obtained. Using these two distributions, we evaluate scores representing the degree of satisfaction of the sufficient conditions for a Nash equilibrium (a situation in which a player will continue with their chosen strategy without incentive to deviate from it). These evaluations show that the non-cooperative admission process can be up to 100% unfair when applicants submit very few applications and up to 60% unfair when each applicant is allowed applications to 10 different graduate programs. In the next part, we model a simple cooperative admission process where the (3x) choices of an applicant are reduced optimally to (x) universities using currently available information

on applications. We see a decrease in the conflict between selected and non-selected candidates, making the process fairer even for fewer applications per student.

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Part I

Introduction

Chapter 1

Introduction and Literature survey

Ranking universities is a valuable indicator for students, academics, university administrators, industry, and government. Teaching, research, citations, worldwide orientation, industry income, and other criteria influence global university rankings. The global university rating is usually declared once a year. For numerous reasons, it is necessary to examine the upcoming rank of a specific university before broadcasting the current year's rating. Depending on anticipated rankings, prospective students need this to apply to particular universities. University officials should determine the varsity's upcoming ranks to strengthen specific fields and raise their standards compared to others. Industry and government should endeavour to estimate university rankings to provide subsidies to the most qualified institutions in the following year. Data analysis and machine learning concepts and techniques [1], [2] and [3] are extremely useful in explaining the past and forecasting the future by analysing and exploring data.

Ranking tables for universities are created by designers and publishers with the goal of objectively measuring each university's excellence. To achieve this, they typically collect data that they believe indicate quality and assign a predetermined weight to each parameter. A university's ranking is then determined based on their overall score. However, there are noticeable variations in the methods and metrics used across different ranking tables [4].

Comparative analysis of several university ranking systems can provide essential knowledge for a wide variety of interested users to better comprehend the information offered in these systems and interpret and apply it in an informed, responsible manner. Different approaches are used in various national and international university rating systems. We picked two of the most influential ranking systems, THE World University Ranking and QS World University Ranking. Both THE and QS ranking systems provide performance indicators that justify research-intensive universities across all their prime objectives. THE Ranking system uses 13 performance indicators divided into five categories Teaching, Research, Citations, International Outlook and Industry income. QS Ranking universities are evaluated based on six metrics Academic reputation, Employer reputation, Faculty-Student ratio, Citations per faculty, International faculty ratio and International student ratio. Some of these parameters affect ranking more than others, and their analysis will be done in the methodology section.

Many academics have criticised the ranking method. Marginson's research [5] shows the flaws in balancing different indicators, which leads to ranking methodology and mistakes. It is frequently necessary to explain why a certain technique or indicator was chosen, how well it was established, who selected it, and how open and reflective the selection process was [6]. As an indication, the Academic Ranking of World Universities (ARWU) ranking table uses scientifically quantifiable, objective data. However, the best universities are classified as science-oriented schools. The Times Higher Education Supplement (THES) ranking table is mostly based on expert and recruiter evaluations of a university's prominence and impact. According to Marginson [5], the THES ranking table hardly touches on the research component while failing to express the education component.

An international ranking group (formed in 2004) brought a set of principles on the ranking of higher education institutions. The principles called Berlin principles are divided into four sets:-

- **Ranking's purpose and objectives**: To evaluate the inputs, processes, and outputs of higher education, the purpose and goals of ranking should be one of many varied techniques; be explicit about their aims and their target audiences.
- **Indicator design and weighting**: The creation and evaluation of indicators be open about the process used to determine rankings, choose indicators based on their applicability and reliability, make the weights given to various indicators clear (if utilised), and restrict modifications to them.
- **Data collection and processing**: When gathering and processing data, it is crucial to employ audited and verified information wherever feasible and information gathered under accepted scientific data collecting practises.
- **Presentation of ranking results**: It should give users a clear understanding of all the criteria considered when creating a ranking, provide them with a choice in how rankings are shown, guarantee that errors in the original data are eliminated or reduced during compilation, and be organised and published in a way that allows for error and fault correction.

The ”most significant common denominators” across groups of rankers with different viewpoints, according to Enserink [4], make the principles highly generic.

Data gathering, which is related to methodology, is another issue with university rankings. Numerous rating lists are constructed using facultative, qualitative data the colleges provide. Peer reviews, in which academic authorities or graduate recruiters evaluate colleges, are much more contentious, according to Enserink [4].

We take a data-based approach similar to that taken by Moed (2017) [7], Robinson 2019 [8], and Selten (2020) [9] for analysis.

The paper by Moed (2017) [7] compares five different rankings and observes that each ranking system is biased towards a particular geographical region, such as ARWU towards North America and Western Europe, U-Multirank towards Europe, QS and THE towards Anglo-Saxon countries, as Great Britain, Canada, and Australia appear on both and LEIDEN towards emerging Asian countries and North America, by simply calculating the ratio of expected and actual institution from a country appearing in Ranking. They also focussed on the top 100 of each ranking system and observed that only 35 universities appear in the top 100 of every other, further supporting the claim.

Moed (2017) [7] also shows that THE ranking system uses a *percentile rank* based approach for all its parameters except the Academic Reputation Survey for which *cummulative probability function* is calculated and also an exponential component is added to it, by plotting scores in THE Ranking against percentiles rank scores calculated by author. THE Reputation parameters score deviates from percentile rank scores. In contrast, other parameters closely follow it, and due to its exponential component, only 10% of institutions have a Research or Teaching Performance Score above 55 or 50, respectively. Also, when considering all five rankings highest skewness comes for ARWU and THE Reputation parameter, while the lowest skewness is observed for QS Reputation parameters. This observation shows that QS uses *Regional Weightings* to counter any discrepancies in the data. This is also supported by finding the Pearson correlation coefficient between citation-based indicators from Leiden, THE, ARWU, and U-Multirank, which show robust rank correlations with one another but correlate only weakly with the QS Citation per Faculty indicator.

The paper by Robinson (2109) [8] performs PCA on seven rankings with h-index (Indicator at the institutional level. Therefore, a university will have an h-index of h if it has at least h of its publications have received at least h citations). The first component extracts 93% of the variance of the H-index, which is close to the first component of each ranking's first component since ranking and H-index are highly correlated. The high correlation of the H-index with other ranking shows that the one-dimensional character of the ranking dataset is related to publication output and citation impact.

The paper by Selten (2020) [9] conducts a longitudinal analysis of the ranking system for 2012-2018, mainly for ARWU, THE, and QS ranking. The initial research shows that when comparing the top 100 of the same ranking system, there is not much change in the ranking positions of overlapping universities when rankings for 2013-2018 compared with 2012 ranking data. This analysis shows that rankings are very stable over time. Stability can be explained by the ranking system using rolling averages to measure publications and citations. It can also be due to the Circular effect, reinforcing universities' position since reputation surveys are affected by previous rankings. This paper[9] also analyses regional biases of the ranking system by plotting Research performance against Reputation performance by categorizing universities based on region and the grounds of the language spoken separately for both. The analysis was that English-speaking universities like the US and UK are high on the Reputation and citation scale in ARWU. In THE and especially QS ranking, Asian universities are performing well and climbing fast on the research performance scale. In QS, even when the research output was low in the initial years, the reputation was high; this shows that QS ranking supports Chinese universities.

The second part of the research is concerned with modelling. Most of the studies and research are conducted for modelling the admission process using a data analysis approach and machine learning-based models [10, 11, 12], which considers the student's profile and using data analysis and machine learning models it predicts the chances of student's chances of getting admission in a particular university. Our approach differs from the traditional method and is concerned with modelling the admission process (both non-cooperative and cooperative type) for a large number of students using the Sequential Monte Carlo method [13, 14] and determining how the number of applications received and the number of applications per student affect the fairness of the admission process.

While drawing on this earlier work for inspiration, we go beyond it by graphically performing our analysis across two rankings (QS and THE), using various analytical techniques and geographic and sample comparisons.

In the first part of this work, we studied the university ranking systems and their pitfalls. The dominance of two indicators in most rankings, i.e. citations per faculty and academic reputation, was shown. Using correlations between different ranking systems, and correlations within a ranking system for an offset of 2 yrs, it was highlighted that the ranks $\sim>$ 100 are unreliable. Next, using the correlation among these two parameters, the geographical bias in a well-known university ranking (QS) was demonstrated using Chi-square tests. Sensitivity tests on these rankings were also performed to show that a 25% noise in the score represents a 10% change in rankings.

In the work's second part, we model the graduate admission process in US universities, where the above rankings play a crucial role. We model the competitive non-cooperative application process (where people who take the test are unaware of the distribution of scores for the exam involving all the candidate's scores, the academic performance and the choice of universities of other applicants is also unknown) to determine the effects of conflict among applications to different programs on the fairness of the admission process. Each applicant can apply to a small set of graduate programs in various universities, which amounts to a considerable sum of money every year. This process was modelled using a Sequential Monte Carlo [13, 14] approach with a normally, uniformly and exponentially distributed population of merit scores, where the final distribution of the admitted and declined students after the randomized process was obtained. Using these two distributions, we evaluate scores representing the degree of satisfaction of the sufficient conditions for a Nash equilibrium (a situation in which a player will continue with their chosen strategy without incentive to deviate from it). These evaluations show that the non-cooperative admission process can be up to 100% unfair when applicants submit very few applications and up to 60% unfair when each applicant is allowed applications to 10 different graduate programs. In the next part, we model the cooperative admission process (where each applicant tries to optimize the result of the admission process). We see a decrease in the conflict between selected and non-selected candidates, making the process fairer even for fewer applications per student.

In Chapter 2, We are analyzing Ranking parameters and pitfalls of ranking. We use various statistical parameters and methods like the Pearson correlation coefficient, Chi-squared test, and sensitivity analysis for this.

In Chapter 3, We are trying to model the non-cooperative admission process using a stochastic model. We use the 'Sequential Monte Carlo method to model the admission process and analyze the conditions when the merit scores are highly correlated to the probability of success in gaining admissions (i.e., a sufficient prerequisite for a weak Nash equilibrium).

In Chapter 4, we are modelling the cooperative admission process. We aim to reduce the cost incurred for students to apply to universities and optimize the selection using 'rejection sampling'. While also weakly satisfying the goal for universities, reducing the conflict between admitted and non-admitted students in the admission process.

Part II

Methodology

Chapter 2

Rankings and analysis of its limitations

In this section, we are analyzing QS and THE ranking system and their shortcomings. We chose these rankings because these are the most commonly used rankings, and datasets for these rankings were readily available for these rankings. Both of these rankings use different indicators and methodologies to measure the performance of universities. The weightage and indicators these rankings use are discussed in the following sections.

2.1 THE world university ranking indicators

The data for THE world university ranking is collected by data scrapping from its website [15].THE uses data collected from Elsevier [16] for its reputation survey parameter. THE world university ranking uses 13 indicators which are combined into five categories Teaching (30%), Research (30%), Citations (30%), International outlook (7.5%), Industry income (2.5%). The distribution and explaination of indicators are as given below.

2.1.1 Teaching

1. Reputation survey (15%)
   (a) Perceived prestige of the institution.
   (b) Non-zero values are standardized.
2. Academic staff-to-student ratio (4.5%)
   (a) Defined as:-

$$\frac{Total\ full\ time\ equivalent\ (FTE)\ number\ of\ staff\ employed}{FTE\ number\ of\ students}$$

   (b) The score is normalized after the calculation.

3. Doctorates awarded to bachelor degrees awarded ratio (2.25%)
4. Doctorates-awarded-to-academic-staff ratio (6%)
   (a) Defined as:-
   $$\frac{Total\ subject\ weighted\ doctorates}{Total\ subject\ weighted\ number\ of\ academic\ staff}$$
   (b) Takes into account unique subject mix.
5. Institutional income per staff (2.25%)
   (a) This metric is generated by dividing the institutional income adjusted to PPP, by the total number of academic staff.

2.1.2 Research

1. Reputation survey (18%)
   (a) Perceived prestige of the institution
   (b) Non-zero values are standardized
2. Research income per staff (6%)
   (a) Defined as:-
   $$\frac{Total\ subject\ weighted\ research\ income\ adjusted\ for\ PurchasePowerParity\ (PPP)}{Total\ subject\ weighted\ number\ of\ academic\ staff}$$
   (b) This indicator takes account of each institution's distinct subject profile.
3. Research productivity (6%)
   (a) This metric is given as:-
   $$\frac{Total\ subject\ weighted\ number\ of\ papers\ published\ in\ the\ academic\ journals}{The\ sum\ of\ the\ total\ subject\ weightednumber\ of\ FTE\ research\ staff\ and\ FTE\ academic\ staff}$$

2.1.3 Citations

1. Calculated using the average number of times a university's published work is cited globally by scholars.
2. Data is normalized after calculation to avoid disparity between different subject areas.

2.1.4 International Outlook

1. Proportion of International students (2.5%)
   (a) Calculated as:-
   $$\frac{Total\ FTE\ number\ of\ international\ students}{Total\ FTE\ number\ of\ students}$$
2. Proportion of international staff (2.5%)
   (a) Calculated as:-
   $$\frac{Total\ FTE\ number\ of\ international\ academic\ staff}{Total\ FTE\ number\ of\ academic\ staff}$$
3. International collaboration (2.5%)
   (a) Calculated as:-
   $$\frac{Total\ subject\ weighted\ number\ of\ publications\ with\ at\ least\ one\ international\ co-author}{The\ total\ subjected\ weighted\ number\ of\ publications}$$

2.1.5 Industry income

1. Institute's ability to attract funding in the commercial marketplace.
2. Calculated as:-

$$\frac{Research\ income\ an\ institution\ earns\ from\ industry\ (adjusted\ for\ PPP)}{Total\ number\ of\ FTE\ academic\ staff\ it\ employs}$$

2.2 QS world university ranking criteria

QS world university ranking [17] uses six indicators which are as follows academic reputation (40%), employer reputation (10%), faculty-student ratio (20%), citations per faculty (20%), international faculty ratio (5%) and international student ratio (5%). The distribution and explanation of indicators are given below.

2.2.1 Academic and Employer Reputation

1. Weighted counts of international nominations.
2. Weighted count of domestic nominations.
3. Both domestic and international count is normalized and combined.
4. Different transformations technique is applied to minimize the impact of outliers.

2.2.2 Faculty-Student Ratio

1. Defined as:- $$\frac{Total\ full\ time\ equivalent\ (FTE)\ number\ of staff\ employed}{FTE\ number\ of\ students}$$

2. The score is normalized after calculation.

2.2.3 Citations per faculty

1. It considers the relative intensity and volume of research being done at an institute.

2. Data is normalized after calculation to avoid disparity between different subject areas.

2.2.4 International faculty ratio (IFR) and international student ratio (ISR)

1. IFR calculated as:-

$$\frac{Total\ FTE\ number\ of\ international\ academic\ staff}{Total\ FTE\ number\ of\ academic\ staff}$$

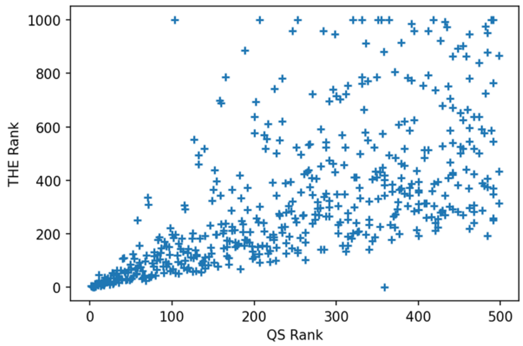
2. ISR calculated as:- $$\frac{Total\ FTE\ number\ of\ international\ students}{Total\ FTE\ number\ of\ students}$$

2.3 Goals and Highlights

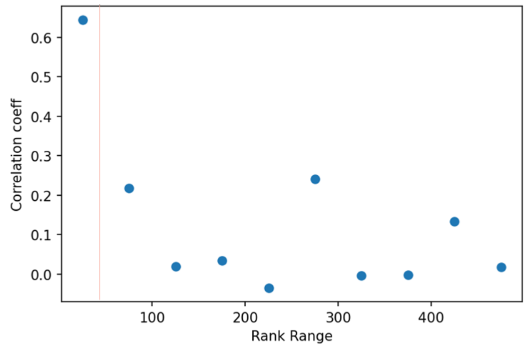
The goals and highlights of the project can be divided into three parts.

2.3.1 To show that the ranking systems are unreliable for lower ranks

An initial comparison is made between QS and THE Ranking for the year 2020 in Fig 2.1a, which shows that the correlation between the rankings is high for the initial 50 ranks only after that correlation between both decreases, as seen in Fig 2.1b. For QS ranking, when compared for two different years, 2018 vs 2020, while the initial correlation is much higher when compared with the correlation for two different rankings with each other, the decrease in correlation is also smaller, as can be seen from Fig 2.2a and Fig 2.2b. This shows that the same ranking is stable and doesn't change much over the years. We also compared NIRF ranking for two years gap. The overall correlation is high 2.3a for

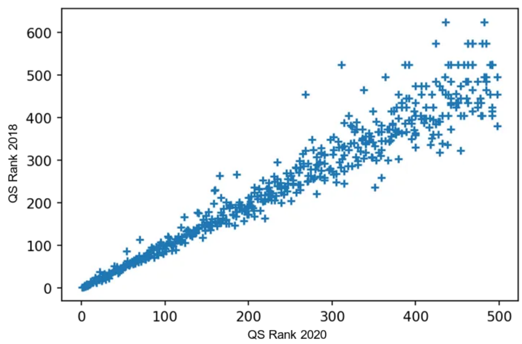


(a) QS vs THE (correlation=0.5343) comparison for different types of ranking are loosely correlated with each other.

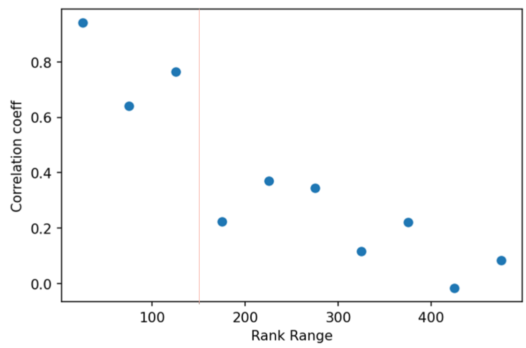


(b) Corr. Coeff. Per bin (size=50) shows that correlation is moderate till top 50 ranks and after that correlation is very low for lower ranks.

Figure 2.1: QS vs THE ranking (2020)

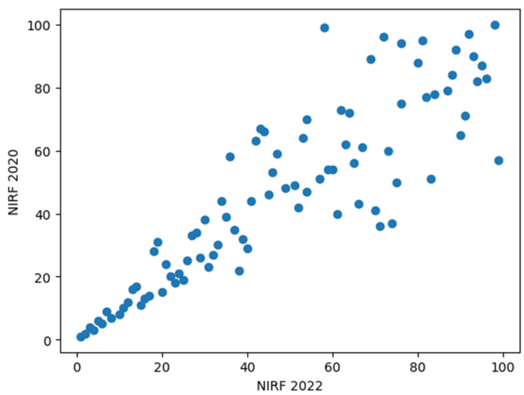


(a) 2018 vs 2020 (correlation=0.9054) QS ranking comparison shows high correlation with that of different ranking, but it breaks down at lower ranks.

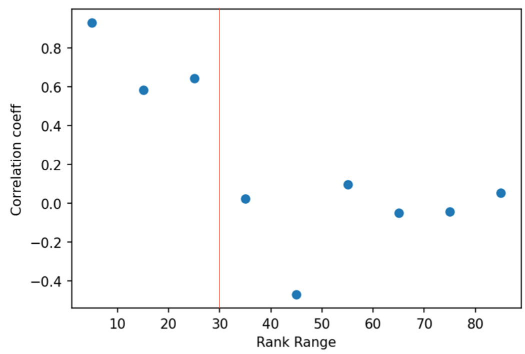


(b) Corr. Coeff. Per bin (size=50) shows a high to moderate correlation to the top 150 ranks and a weak correlation for lower ranks.

Figure 2.2: QS ranking 2018 vs 2020



(a) 2020 vs 2022 (correlation=0.9303) NIRF ranking comparison shows high overall correlation when compared with that of different ranking.

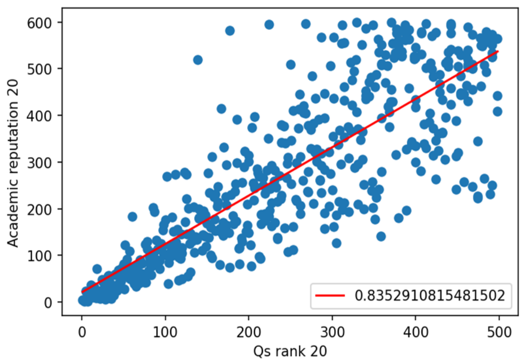


(b) Corr. Coeff. Per bin (size=10) shows a high to moderate correlation to the top 30 ranks and a weak correlation for lower ranks.

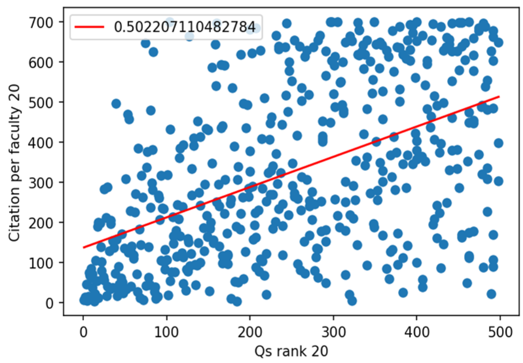
Figure 2.3: NIRF ranking 2020 vs 2022

the top 30 ranks only and nearly decreases to zero afterwards 2.3b.
Pearson's correlation coefficient (Eq 2.1) is used for finding correlation in the data.

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} \tag{2.1}$$



(a) Academic reputation vs QS Rank (correlation=0.8352)



(b) Citation per faculty vs QS Rank (correlation=0.5022)

Figure 2.4: To find out the most important parameter used by ranking, the Pearson correlation coefficient is used, which gives us the two most important parameters used for further analysis of geographical bias.

Further analysis is only done for QS since THE ranking starts giving range after 200 ranks. The global QS university rankings data set consists of 6 parameters in the overall scores known as performance indicators of the university: Academic reputation, Employer reputation, Staff to student ratio, Citation per faculty, international faculty, and student ratio. A simple Pearson correlation coefficient determines the two most important features. Our analysis yielded Academic reputation (Fig 2.4a) and Citation per faculty (Fig 2.4b) as the two most important factors with high correlation coefficients affecting the overall scores of universities.

2.3.2 To highlight the Geographical bias of rankings

Geographical bias is obtained using the two most important parameters of ranking, i.e. Academic reputation (AR) and Citation per faculty (CPF). We clustered the universities into four groups for each region:

- AR > mean and CPF > mean
- AR > mean and CPF < mean
- AR < mean and CPF > mean
- AR < mean and CPF < mean

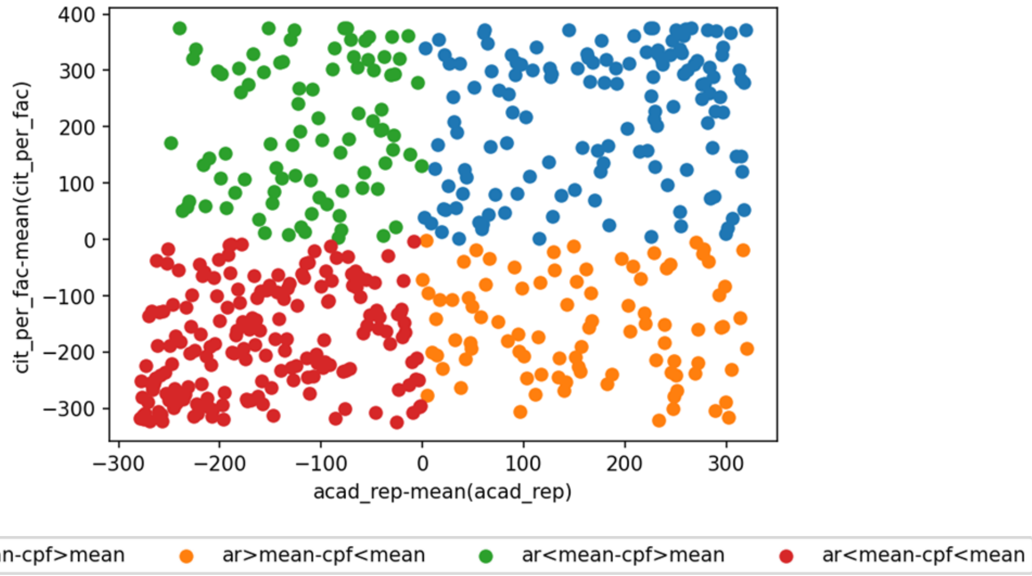


Figure 2.5: Clustering of universities based on Academic reputation and citation per faculty parameter to find geographical bias.

To find out the bias of ranking towards a particular region, we compared the orange dots (ar>mean and cpf<mean) with the green dots (ar<mean and cpf>mean) and orange dots with all (probability of +ve bias) (Table 2.1). We observed that North America, China, Japan and Oceania had a higher likelihood of positive bias than other areas like Europe, Asia etc. The bias (orange/green) for these regions was also higher. This shows that even though universities in these regions have low research output, their rank is higher due to the academic reputation parameter which is based on Elsevier's reputation survey only.

Orange/All		**Orange/Green**	
Europe	0.1690	Europe	0.8780
North America	0.2314	North America	2.2727
China and Japan	0.2682	China and Japan	1.5714
Asia*	0.1851	Asia*	0.8823
Oceania**	0.2058	Oceania**	7.0
Africa	0.25	Africa	-
South America	0	South America	0

Table 2.1: Comparison between orange (AR >mean and CPF <mean) and green (AR <mean and CPF >mean) region universities (QS 2020), where ORANGE/ALL is the probability of +ve bias and ORANGE/GREEN is bias. North America, China, Japan, and Oceania show the highest probability of positive bias.

To further analyze the dependence or independence of QS World University ranking on the geographical region, we performed the chi-square test [18] for independence for all four quadrants for each geographical area separately. For the test, we formed our hypothesis as given below:-

- X_0 (null) - QS Rankings is independent of geographical region
- X_1 - QS Rankings is dependent of geographical region

Geographical region	Blue		Orange		Green		Red		$X^2 = \sum \frac{(o-e)^2}{e}$	X^2 (in percent)
	o	e	o	e	o	e	o	e		
Europe	66	61	36	40	41	38	70	74	1.2628	13.24 %
North America	20	31	25	21	11	19	52	37	14.114	99.306 %
China and Japan	8	12	11	8	7	7	15	14	2.5297	36.07 %
Asia	30	23	15	15	17	15	19	28	5.29	74.12 %
Oceania	9	10	7	6	1	6	17	12	6.5167	83.63 %

Figure 2.6: Chi-squared test to find out independence/dependence of QS Rankings with geographical region shows that North America (99.306%), Asia (74.12%) and Oceania (83.63%) have the highest X^2 value, therefore, rejecting the null hypothesis and indicating QS Rankings is biased towards these regions (Note that Asia is affected by a negative bias, while North America and Oceania seem to enjoy a positive bias).

For each region, the expected and observed university count is given. Using the chi-square test statistic, we get chi-square summation values and, thereby, chi-square value in percent. Table 2.6 concludes that the null hypothesis can be rejected at 5,20, and 25 percent significance levels for North America, Oceania and Asia. This shows that our preliminary assumption that QS ranking is independent of the geographical region is false.

2.3.3 To determine the sensitivity and robustness of the ranking system

The above task (Fig 2.7) in investigating robustness is more straightforward and lends itself to a more precise characterization [19]. Given $\hat{r}$, a n-dimensional vector containing natural integers 1 to n representing n universities in the list may be used to characterise their ranks. Let $\Delta\hat{r}$ be the change in the ranking vector caused by a difference in the scoring formula's inputs. We want to assess the dependability of the rankings by calculating a vector $\Delta\hat{r}$ with the highest l_1-norm, given the size of projected changes in the scores F. The l_1-norm $\|.\|_1$ implies that the length of the vector $\Delta\hat{r}$ representing the change in ranking list is given by the sum of absolute values of the entries, i.e. $\sum_n |\Delta r_i|$, and the

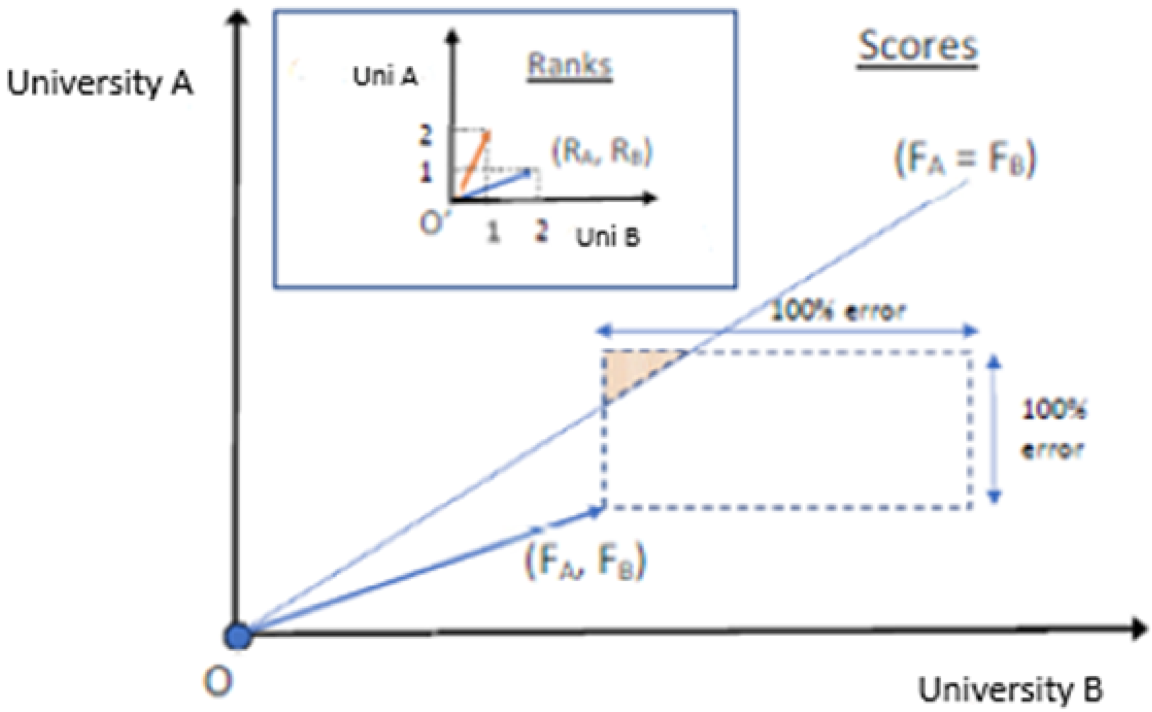
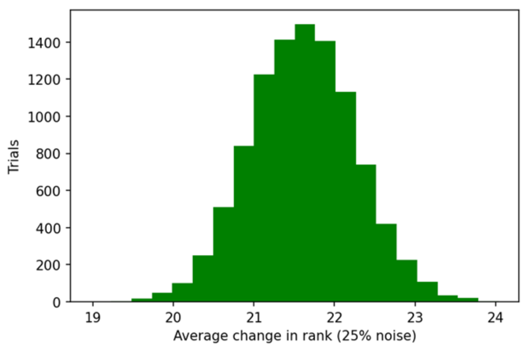
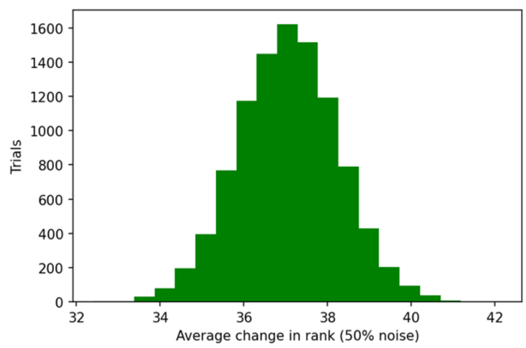


Figure 2.7: The problem of sensitivity and robustness can be explained using the score value of two institutions represented by (F_A and F_B). When there is up to 100% error in the scores calculation of these two universities, a change in the ranking vector can occur, which can be traced through all possible score changes in the rectangle. When we consider this problem for n-universities, then this 2-d box is converted to an n-d box with the most significant change in the ranking list represented by one of the corners. Performing random sampling in the box can help us determine the most considerable change in rankings possible. [19].



(a) 0-25% errors



(b) 0-50% errors

Figure 2.8: Average change in rank for 10^4 trials.

maximum value of $\|\Delta\hat{r}\|_1/n$ is the maximum possible (average) change in the rank of all the n universities in the list.

Variation of input scores by introducing up to 25% (Fig 2.8a) and 50% (Fig 2.8b) noise for 10^5 trials resulted in an average change in rank of around 18-24 for 25% and 33-41 for 50% rank which shows quite a bit of instability in the ranking system.

To analyze the sensitivity of the ranking system for a particular rank, we calculate the change in rank for each rank averaged over the number of iterations (i.e. $\frac{\Delta\hat{r_1}+\Delta\hat{r_2}+.....+\Delta\hat{r_i}}{i}$).

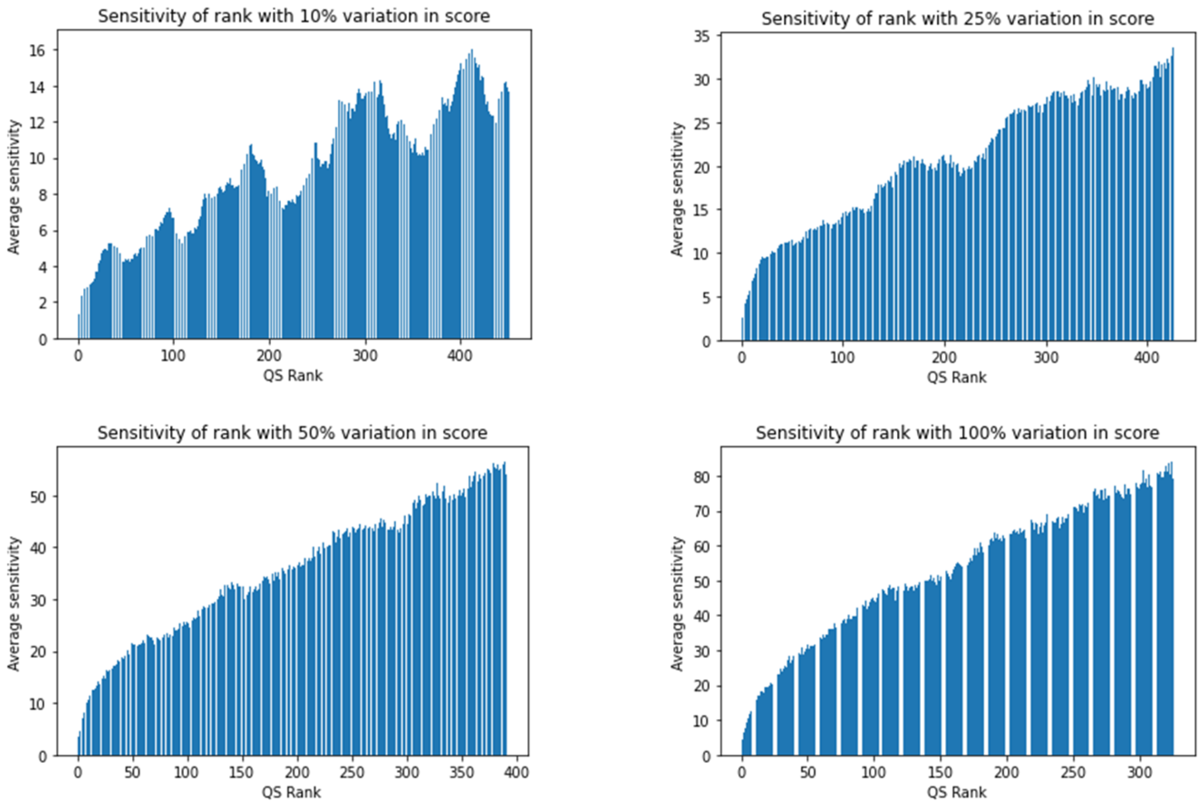


Figure 2.9: The change in ranking owing to 0-100% errors at the original scores in the list reflects the average sensitivity.

For sensitivity analysis, adding noise to input scores at different levels yielded similar results, with higher noise levels increasing average sensitivity for low-rank regions and relative stability in high-rank regions. It is observed from Fig 2.9 that the ranking system is sensitive and unstable for lower ranks and is only suitable for the top 100-150 ranks.

Chapter 3

A stochastic model for non-cooperative admission process

In this section, our goal consists of two parts:-

- Assuming a non-cooperative process, try to model the chances of selecting students based on their scores and generate a stochastic particle model using the sequential monte carlo method [20].
- Identify the conditions when the merit scores are highly correlated to the probability of success in gaining admissions (i.e., a sufficient prerequisite for Nash equilibrium [21]).

Symbols	**Explanation**	**Symbols**	**Explanation**
N	No. of students	n	No. of universities
C	n universities cut-off marks	m	No. of application per student
g_1	Universities with high cut-off scores	g_2	Universities with mid cut-off scores
g_3	Universities with low cut-off scores	C_i	i^{th} University's cut-off marks
S_i	i^{th} Student's marks	Y	Open seats per university

Table 3.1: Model parameters and their explanation

Initially, we were given cut-off scores for 500 universities; we generated sigmoids for each university based on cut-off marks as the parameter, then stored sigmoids in a dictionary with decreasing order of cut-off marks and index as keys and sigmoids as values.

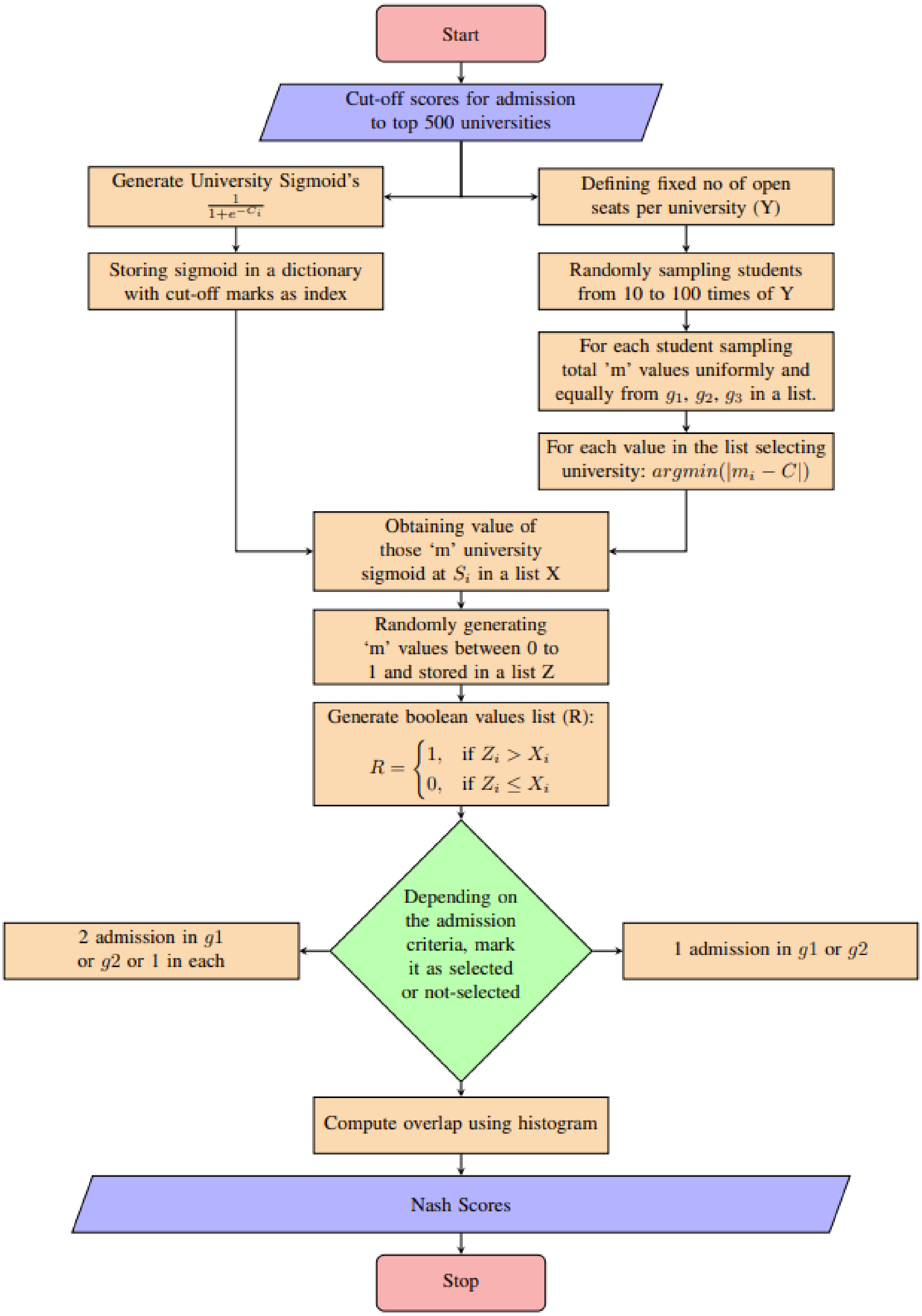


Figure 3.1: Flowchart of the stochastic model

Activity from Student's side	Activity from University's side
• Each student is sampled from the population's normal distribution of merit scores. • Student chooses 'm' universities based on his merit scores. • Cut-off of each university is related to it's ranking. • For each student, the 'm' universities are chosen randomly from three groups, g1, g2 and g3 which are defined according to student's merit score.	• Universities receive applications from all the students. • Once applications are received, they go through a random process of being accepted or not accepted in the merit list. • For each university, we apply a sigmoid based on its cut-off score, obtain its value at each student's marks, and compare it with a randomly generated probability value. • Based on the available open seat and the result of the above comparison, they pass or fail the admission process. • Once accepted, they have a merit list in the order of merit scores.

Figure 3.2: Overview of the non-cooperative stochastic process

We define a fixed number of open seats per university (Y), then randomly sample students 2 to 75 times 'Y' from a normal, exponential, and uniform distribution. For each student, defined g_1,g_2, and g_3 represent the range in which universities with a cut-off score (C_i) have a higher, equal and lower value than the student's marks (S_i) score lies. Then for each student, uniformly and equally sampling 'm' values from g_1,g_2, and g_3 combined and stored them in a list. Now for 'm' values in the list, we find 'm' universities closest to those values using '$min\_index = argmin(|m_i - C|)$', where m_i is the i^{th} value in the list and C is the cut-off marks list for all 500 universities.

It returns the index (key) of the university where the function attains a minimum value. We obtain the university sigmoids and their value at i^{th} student's marks (S_i) using that key. We do this for all 'm' universities, store these values in a list, and then compare each value with a randomly generated value between 0 and 1 using Equation 3.1.

For each 'i',

$$R = \begin{cases} 1, & \text{if } Z_i > X_i \\ 0, & \text{if } Z_i \leq X_i \end{cases} \tag{3.1}$$

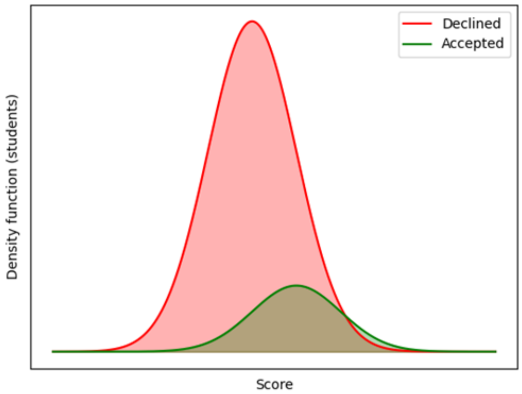
Where,

Z_i $= i^{th}$ randomly generated value

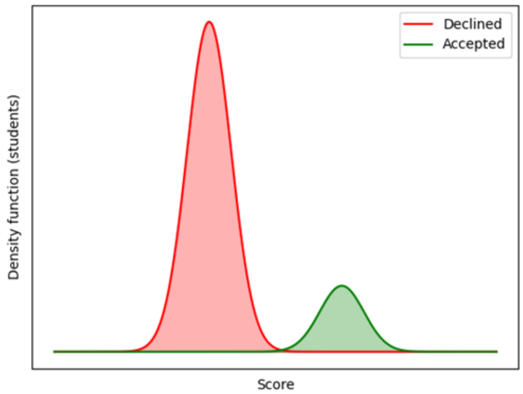
$X_i = i^{th}$ the value obtained from university sigmoid

Here R is a list containing acceptance and rejection of admission in g_1,g_2, and g_3 in boolean values. Depending on the admission criteria and no. of unfilled open seats, we store the student as selected or rejected. After that, we used two types of Admission criteria:

- One entry in g_1 or g_2.
- Two entries in g_1 or g_2 or one in each.



(a) High overlap between selected and rejected resulting in nearly 100% Nash score



(b) Low overlap between selected and rejected resulting in Null Nash score

Figure 3.3: Nash scores calculation criteria

After receiving the list of accepted and rejected students, we calculate the overlap (Fig. 3.3) between these two using equation3.2 and obtain Nash score and repeat the process for a different number of students. Fig 4.1 flowchart explains the above process step by step.

$$NS = \int f_1 * f_2 \, dx \tag{3.2}$$

Where,

NS = Nash score
f_1 = distribution of scores for selected students
f_2 = distribution of scores for rejected students

The above process can be implemented using the algorithm (Algorithm 2) for a fixed number of students. It can be repeated for different numbers of students per open seat to obtain the Nash scores matrix, which can then be used to generate the colour map of Nash scores.

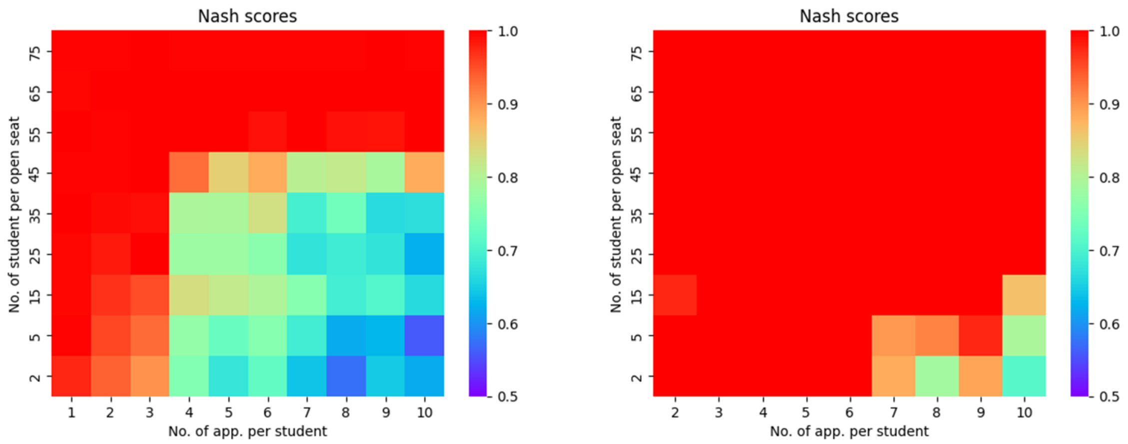
3.1 RESULTS

After the list of cut-off scores was obtained from our model for selected and non-selected candidates, we computed the Nash scores using equation 3.2. Since the actual data was not available to model the different types of students distributions possible and to make our results as generalised as possible, we are generating student's scores from the following three distributions:-

Notation	Explanation
SPO (Y-axis)	Number of students per open seat (Total students (N)/Total seats)
m (X-axis)	Number of applications per student

Table 3.2: Color Map axis Notation and Explanation

3.1.1 Normal distribution



(a) One admission in $g1$ or $g2$

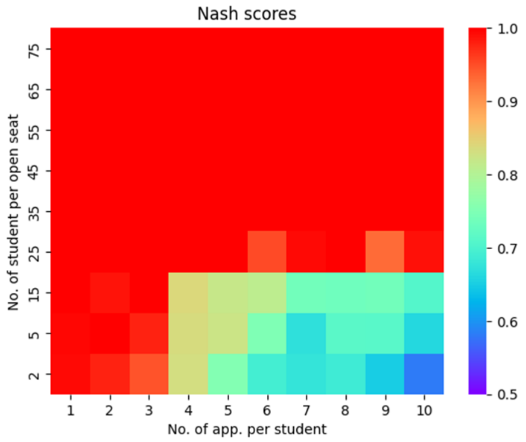
(b) Two admission in $g1$ or $g2$ or one in each

Figure 3.4: Nash scores for normal distribution

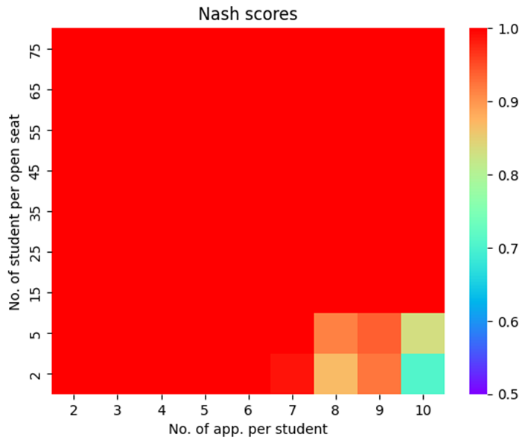
In this case, our exams score is normally distributed. It is the most common case of distribution of scores possible.

In Fig 3.4a and Fig 3.4b, we can see that when the number of students (high SPO) is much greater than the number of universities and the number of applications per student (m) is much less than the number of universities, the selection process becomes unfair, because students with lower scores are getting selected, in contrast, those with higher scores might not get selected due to this there is high overlap between selected and rejected. When we changed the criteria for selection to a minimum of two admission cases, the process became even unfairer for a lower number of students per open seat. While if we increase increase 'm', then we see an improvement in the Nash score, and the process becomes a bit fairer since increasing 'm' ensures that students with higher scores will not get rejected for admission.

3.1.2 Uniform distribution



(a) One admission in $g1$ or $g2$



(b) Two admission in $g1$ or $g2$ or one in each

Figure 3.5: Nash scores for uniform distribution

The uniform distribution case is when only the top scorers apply for the admission process, like a flatter region of normally distributed scores.

In uniform distribution, we observed that the process becomes even unfairer compared to normal distribution for the same *SPO* parameter. When the uniform distribution is compared to normal, it is visible that Nash score is nearly 100% when *SPO* is above 25-30 but for normal, it is around 45-50, when the 'accepted application set' is a set with one admission. A similar trend is observed when the 'accepted application set' is set with two admissions, and the process is 100% unfairer in normal distribution cases for *SPO* above 20-25 and above 5-10 for uniform distribution.

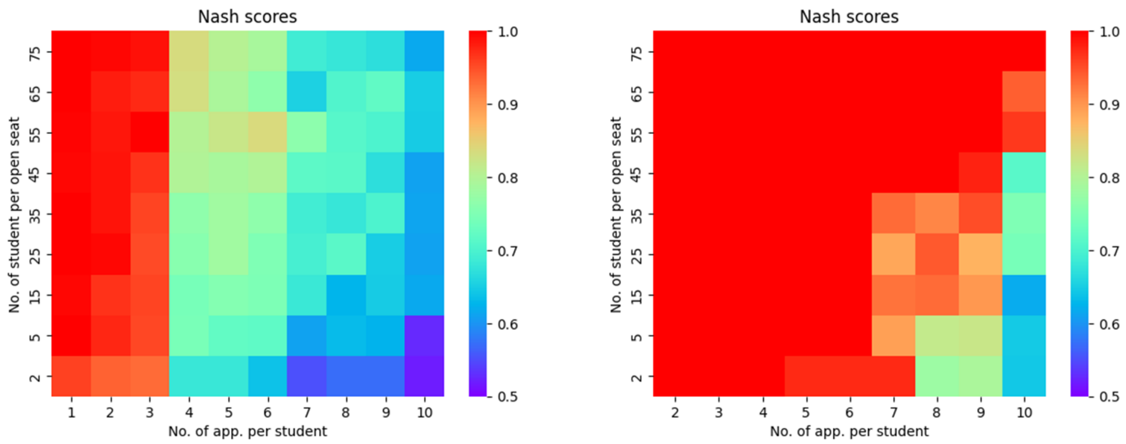
When comparing the m parameter for both distributions, the trend observed in uniform distribution is similar to that of normal distribution. The process becomes fairer for a higher m. Also, Increasing the number of accepted applications from one to two requires increasing m to achieve equal fairness.

This shows that when scores are normal, the non-cooperative process tends to be fairer than when scores are uniform since, in a normal distribution, students in the high-score region are less compared to a uniform distribution so there is less overlap (Fig. 3.3b) between selected and rejected students since the selected curve is shifted towards the right while rejected curve shifts towards the left.

3.1.3 Exponential distribution

The exponential distribution represents when the exam is too tough, and most students only lie in the low score region.

In exponential distribution, when the accepted application set is a set with at least one admission, the process saturates at nearly a constant Nash score for each fixed no of applications per student. While in the case when the accepted application set is a set with at least two admissions, the Nash score increase and the process becomes unfairer (Fig 3.6b). Of all three distributions, exponential had the least Nash scores (overlap between selected and not selected) since the student with high scores were less in exponential, followed by normal and then uniform.



(a) One admission in $g1$ or $g2$

(b) Two admission in $g1$ or $g2$ or one in each

Figure 3.6: Nash scores for exponential distribution

Chapter 4

A stochastic model for cooperative process

In this chapter, our goal is:-

- Receiving the choice of the universities from students and using rejection sampling to optimize and reduce the choice by three times to save the cost incurred for more number of application.
- Secondary goal is for universities to reduce the conflict between admitted and non-admitted students.

Symbols	**Explanation**	**Symbols**	**Explanation**
N	No. of students	n	No. of universities
C	n universities cut-off marks	m	No. of application per student
g_1	Universities with high cut-off scores	g_2	Universities with mid cut-off scores
g_3	Universities with low cut-off scores	C_i	i^{th} University's cut-off marks
S_i	i^{th} Student's marks	Y	Open seats per university

Table 4.1: Model parameters and their explanation

In the previous chapter, our work consisted of stochastic modelling of the admission process for students, in which sampling criteria of m values are done by forming three regions g_1,g_2, and g_3, which represents the range in which universities with a cut-off score (C_i) have a higher, equal and lower value than the student's marks (S_i) score lies. While in

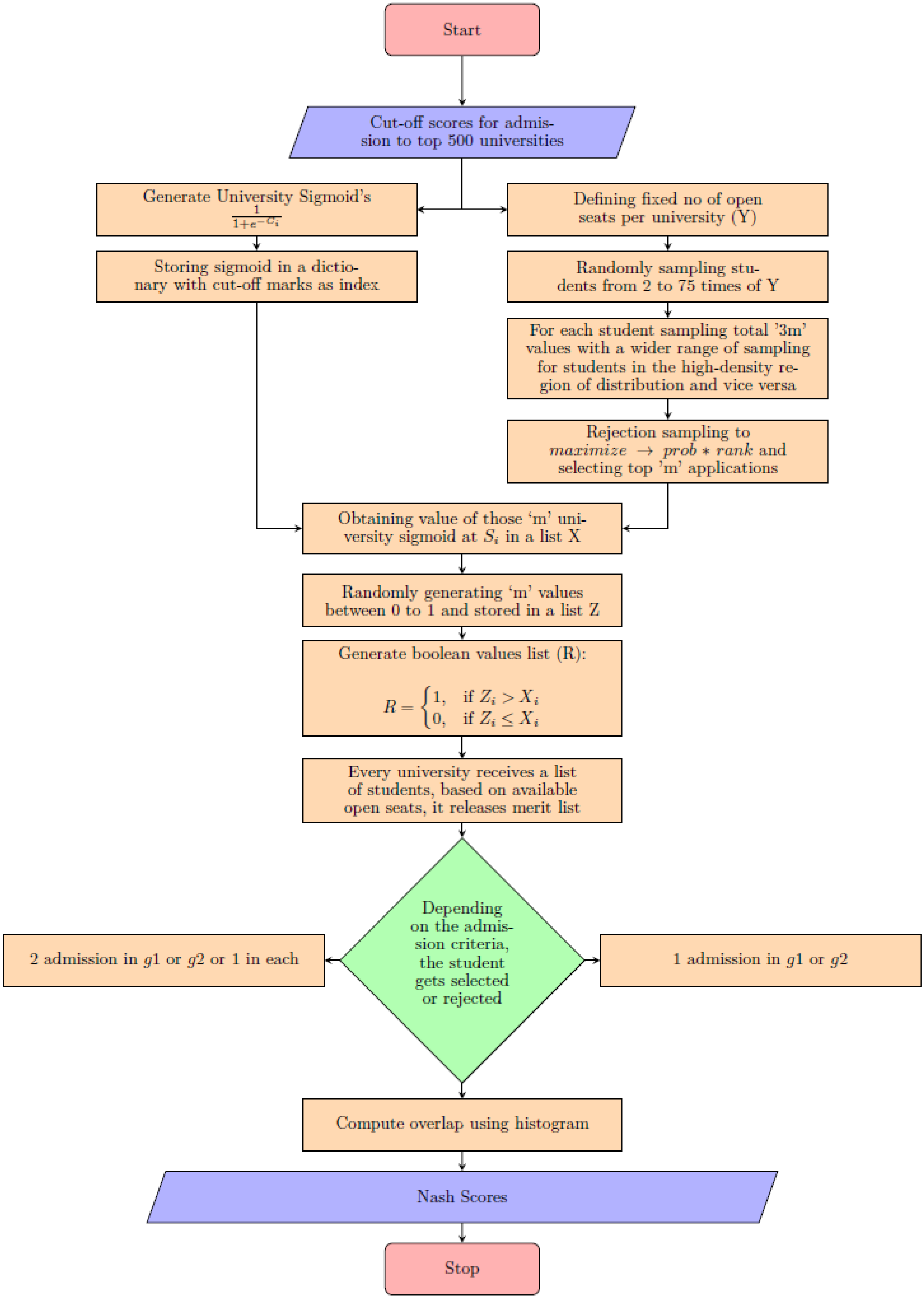


Figure 4.1: Flowchart of the stochastic model for non-cooperative process

Activity from Student's side	Activity from University's side
• Each student is sampled from the population's normal, exponential or uniform distribution of merit scores.	• Universities receive applications from all the students. • Once applications are received, they go through a random process of being accepted or not in the merit list.
• Student chooses '3m' universities with a more comprehensive selection range for universities if their score lies in a higher density region of the distribution and vice versa.	• Based on the available open seat and the result of the random process, they pass or fail the admission process.
• Finally, the '3m' choices are reduced to optimal 'm' choice of applications on the fly using 'Rejection sampling'.	• Once accepted, they have a merit list in the order of merit scores.

Figure 4.2: Overview of the non-cooperative stochastic process

the non-cooperative process, we kept the number of applications per student (m) fixed, whatever the choice of the student from g_1,g_2, and g_3 but in the cooperative process, we are using rejection sampling, which is used to reduce the number of application per student to optimal m choice from $3m$ choices.

In the non-cooperative process (Algo 2 and 1), sample scores were chosen from g_1, g_2 and g_3 distribution based on the student's score. But in the case of the cooperative process (Algo 6), we use the original distribution density curve of all student's scores and vary the range for the selection of scores for a particular student based on the probability density region in which he lies. When a particular student lies near the high probability density region selection range is kept at a maximum. His selection range decreases when a student's score is in a low probability density region. The implementation of the above is done in the function 'select_random_scores'.

The implementation of the function 'select_random_scores' for the cooperative process for normal (Algo 3), exponential (Algo 4), and uniform (Algo 5) is explained here.

In Normal distribution (Algo 3), we receive the student's score as input. Based on the normal density curve of the score distribution, we generate a multiplying factor (equation 4.1) that helps decide the upper and lower scores limit from which a student can pick a university. While in the case of exponential (Algo 4), we use a multiplying factor (equation 4.2) that varies exponentially, and in uniform distribution (Algo 5), we choose a fixed length range since the distribution of scores is uniform.

$$mf = \frac{\mathcal{N}(\mu = mean, \sigma = mean/5)_{at\ S_i}}{\mathcal{N}(\mu = mean, \sigma = mean/5)_{at\ mean}} \tag{4.1}$$

Where,

S_i $= i^{th}$ student's score
$mean$ = mean of the score distribution
mf = multiplying factor

$$mf = \frac{\exp(\sigma = 25)_{at\ S_i}}{\exp(\sigma = 25)_{at\ zero}} \tag{4.2}$$

Where,

S_i $= i^{th}$ student's score
mf = multiplying factor

After receiving '3m' choices of universities from students, we use equation 4.3 to try to find the optimal 'm' choices which optimize the rank of university and probability of selection of a student in a particular university and select those 'm' universities which have the higher value of equation 4.3 from '3m' universities. This method is known as 'Rejection Sampling', where we are suggesting the optimal choice of university in which a student has the highest chance of getting admission and thereby reducing the cost of application for the student and also avoiding unnecessary filling of seats from higher score students to lower ranked universities.

$$T_i = (1/R_i)^{1/2} * (O_i/H_i) \tag{4.3}$$

Where,

R_i $= i^{th}$ universities rank
n = Total universities
H_i = Number of higher score student's in i^{th} university
O_i = Open seats for i^{th} university
T_i $= i^{th}$ university choice score rating

After this, the rest of the process is similar to the non-cooperative process explained in Chapter 3 and Flowchart 4.1.

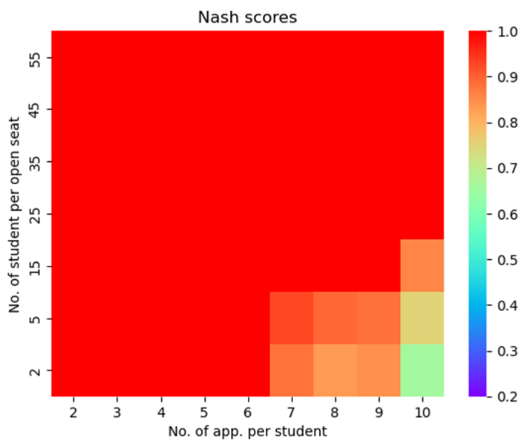
4.1 Non-cooperative and cooperative process results comparison

After obtaining the results for selected and non-selected students, we use equation 3.2 to calculate the area of overlap between both distributions, thereby generating Nash scores. Similar to the non-cooperative process, we obtain the results for different distributions:

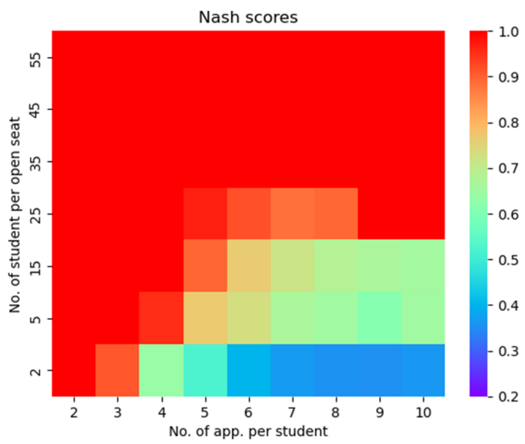
Normal, Exponential and Uniform in the cooperative process also. Below are the results for comparisons between non-cooperative and cooperative processes when the 'Accepted application set' has two admissions:-

Notation	Explanation
SPO (Y-axis)	Number of students per open seat (Total students (N)/Total seats)
m (X-axis)	Number of applications per student

Table 4.2: Color Map axis Notation and Explanation



(a) Non-cooperative process



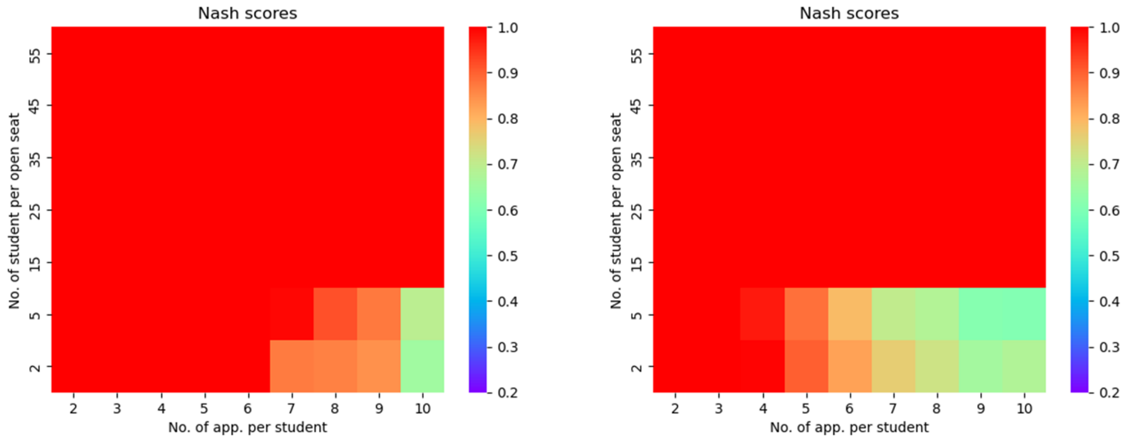
(b) Cooperative process

Figure 4.3: Nash scores for normal distribution in two admission case shows lower improved Nash scores for the cooperative process for lower m, and they scale much better when we increase the SPO parameter.

In the normal distribution cooperative process (Fig 4.3b) case, we see much better results than the non-cooperative process (Fig 4.3a) for lower m, and they scale much better when we increase SPO parameter also. Nash score starts decreasing when m is three and achieves the lowest value of around '0.35' in the cooperative process when SPO is two and m is ten. At the same time, in the non-cooperative process, it starts reducing when m is seven and the lowest value is around '0.7'. For high SPO also, the results are much better in the cooperative case.

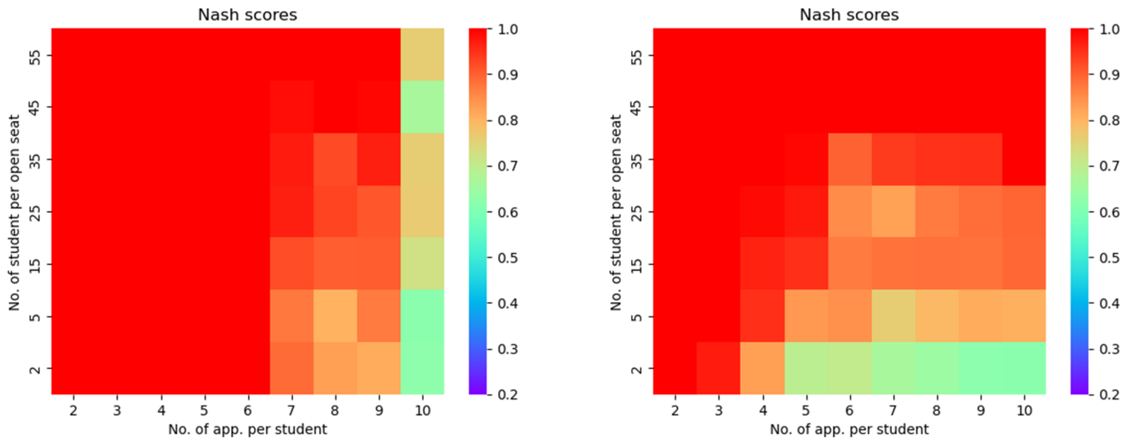
In the uniform distribution 4.4a case, a similar trend is observed to that of normal when we consider the m parameter. However, when we increase the SPO parameter, the scaling observed is not as good as normal but still better than the non-cooperative process.

In the exponential distribution case, the Nash scores are lower for the m parameter in the cooperative process. Still, when m is ten, non-cooperative scores are a bit lower,



(a) Non-cooperative process (b) Cooperative process

Figure 4.4: Nash scores for uniform distribution, a similar trend is observed to that of normal when we consider the m (X-axis) parameter. However, when we increase the SPO (Y-axis) parameter, the scaling observed is not as good as normal but still better than the non-cooperative process.



(a) Non-cooperative process (b) Cooperative process

Figure 4.5: Nash scores for exponential distribution are lower for the m parameter in the cooperative process. Still, when m is ten, non-cooperative scores are a bit lower, similar for higher SPO also. Compared to other distributions, there is less improvement in Nash scores compared to the non-cooperative process.

similar for higher SPO also. Compared to other distributions, there is less improvement in Nash scores compared to the non-cooperative process.

In all three cases, the cooperative process can improve Nash scores for lower m in the order Normal, then uniform and exponential. Lower Nash scores mean lower conflict in selected and not selected students (3.3b). This proves that our suggested cooperative process method can help students save money by optimizing students' choice of universities using 'Rejection Sampling'.

Part III

Conclusion

Chapter 5

Conclusion

University rankings are needed to provide a benchmark for quality, guide funding decisions, drive institutional improvement, promote internationalisation, and enhance public awareness of higher education institutions. However, there are some downsides and limitations to using rankings to measure the quality of education. As we saw in Chapter-2, rankings are only stable in the top 100 range over the years. When comparing two different rankings, there are no correlations since different rankings use slightly different indicators even though the parameters considered are almost identical. Next, we saw that the rankings have an academic reputation as a parameter that highly correlates with the ranking, and the rankings provider uses it to influence the ranking unfairly. The results obtained from our chi-squared tests also proved the geographical bias of rankings. We also did the sensitivity analysis for the rankings, proving the rankings system's instability in lower-rank regions.

In chapter-3, we built a stochastic model for the non-cooperative process using the sequential Monte Carlo method, which shows that for a 40% fair chance of admission, the student may need to apply to more than ten applications amounting to a substantial amount of money. In this process, students may get into lower-ranked universities while having good credentials and vice versa.

In chapter-4, we suggested a stochastic model for the cooperative process. We introduced 'Rejection sampling' that helped optimise student's choice of universities, ensuring a good choice of university and a high probability of admission for the selected choice. Using this model, we brought down the Nash score values for a lower number of applications per student (m), which helped reduce the application process cost from the student's perspective. This avoided unnecessary filling of seats by high-score students in lower cutoff universities, thereby reducing conflict between admitted and declined students.

Appendix A

A stochastic model for non-cooperative admission process

Algorithm 1 Algo to select scores for the non-cooperative process for each distribution

Input : Student's score (S_i), Number of student per open seat (m)
Output : Random scores in g_1, g_2 and g_3
$count\_application \leftarrow 0$
Set g_1, g_2 and g_3 to **Empty list**
while $count\_application < m$ **do**:
    if $count\_application < m$ **then**
        Add $random\_value \in (0.8S_i, 1.2S_i)$ to **List** g_2
        $count\_application \leftarrow count\_application + 1$
    end if
    if $count\_application < m$ **then**
        Add $random\_value \in (1.2S_i, 100)$ to **List** g_1
        $count\_application \leftarrow count\_application + 1$
    end if
    if $count\_application < m$ **then**
        Add $random\_value \in (0.3S_i, 0.8S_i)$ to **List** g_3
        $count\_application \leftarrow count\_application + 1$
    end if
end while
return g_1, g_2, g_3

Algorithm 2 Algo to compute Nash scores.

Input : Cut-off scores for n-universities (C)
Output : Nash scores for a number of student per open seat

for $i \in m$ **do**:
    $ad\_ind = 0$
    $nad\_ind = 0$
    for $j \in N$ **do**:
        $g_1, g_2, g_3 \leftarrow select\_random\_scores(j, i)$
        ▷ this function uniformly selects random scores lying in g_1, g_2 and g_3 region of a particular student

        $uni\_list\_index, R \leftarrow generate\_result(g_1, g_2, g_3)$
        ▷ this function returns universities indexes nearest to g_1, g_2 and g_3 score values and admission result in those universities in boolean values

        for $k \in uni\_list\_index$ **do**:
            $ind = uni\_list\_index(k)$
            if $R(k)$ is True **and** $max\_seats[ind] > filled\_seats[ind]$ **then**
                $filled\_seats[ind] + +$
            else
                R(k)=False
            end if
        end for

        if True values in R matches admission criteria for g_1, g_2 and g_3 **then**
            $admitted\_list[ad\_ind] = j$
            $ad\_ind + +$
        else
            $not\_admitted\_list[nad\_ind] = j$
            $nad\_ind + +$
        end if
    end for
    add $admitted\_list$ to $overall\_admitted\_list$
    add $not\_admitted\_list$ to $overall\_non\_admitted\_list$
end for
$nash\_score \leftarrow calculate\_nash\_scores(overall\_admitted\_list, overall\_non\_admitted\_list)$
▷ this function returns Nash scores for a fixed number of students per open seat

Appendix B

A stochastic model for cooperative admission process

Algorithm 3 Algo to select scores for the cooperative process (Normal distribution)

Input : Student's score (S_i), Number of student per open seat (m)
Output : Random scores in g_2
$count\_application \leftarrow 0$
Set g_2 to **Empty List**
$mf \leftarrow \frac{\mathcal{N}(\mu=mean, \sigma=mean/5)_{at\ S_i}}{\mathcal{N}(\mu=mean, \sigma=mean/5)_{at\ mean}}$ ▷ $mean$ = mean of student scores
$coeff \leftarrow mf * 50$
$low \leftarrow max(0, S_i - coeff)$
$High \leftarrow min(100, S_i + coeff)$
Add $3m$ random_values $\in (low, high)$ to **List** g_2
return g_2

Algorithm 4 Algo to select scores for the cooperative process (Exponential distribution)

Input : Student's score (S_i), Number of student per open seat (m), bins density (d (list))
Output : Random scores in g_2
$count\_application \leftarrow 0$
Set g_2 to **Empty List**
$mf \leftarrow \frac{\exp(\sigma=25)_{at\ S_i}}{\exp(\sigma=25)_{at\ zero}}$
$coeff \leftarrow mf * 100$
$low \leftarrow max(0, S_i - coeff)$
$High \leftarrow min(100, S_i + coeff)$
Add $3m$ random_values $\in (low, high)$ to **List** g_2
return g_2

Algorithm 5 Algo to select scores for the cooperative process (Uniform distribution)

1: **Input :** Student's score (S_i), Number of student per open seat (m)
2: **Output :** Random scores in g_2
3: $count\_application \leftarrow 0$
4: **Set** g_2 to **Empty List**
5: **Add** $3m$ random_values $\in (0.8S_i, 1.2S_i)$ to **List** g_2
6: **return** g_2

Algorithm 6 Algo to compute Nash scores for cooperative process.

```
Input : Cut-off scores for n-universities (C)
Output : Nash scores for a number of student per open seat
Set total_selected and total_rejected to Empty list
for i ∈ m do:
    Set selected, rejected, temp_selected and temp_rejected to Empty list
    Set uni_app_received to Dictionary of Empty list
    for j ∈ N do:
        choice_list ← select_random_scores(j, i)
        ▷ Based on the type of distribution, this function samples the choice of students and
          returns '3m' university choices

        generate_list_of_applications(S_i, choice_list, uni_app_received, rejected)
        ▷ this function takes '3m' choice of application and reduces to 'm' applications using
          'Rejection sampling' and updates uni_app_received.

        for i ∈ uni_app_received do:
            Sort uni_app_received for i^th university
            for j ∈ i do:
                if j < open_seats_uni then:
                    add j to temp_selected List
                else:
                    add j to temp_rejected List
                end if
            end for
        end for

        Create unique_selected Dictionary with value counts of temp_selected List
        Create unique_rejected Dictionary with value counts of temp_rejected List
        for i ∈ unique_selected do:
            if unique_selected(i) ≥ min_admission_required then:
                add i to selected List
                if unique_rejected(i) ≥ 1 then:
                    delete unique_rejected(i)
                end if
            else:
                if unique_rejected(i) is Null then:
                    add i to rejected List
                end if
            end if
        end for

        for i ∈ unique_rejected do:
            add i to rejected List
        end for
        add selected List to total_selected List
        add rejected List to total_rejected List
    end for
    nash_score ← calculate_nash_scores(overall_admitted_list, overall_non_admitted_list)
      ▷ this function returns Nash scores for a fixed number of students per open seat.
end for
```

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