DS286 | 2016-10-14

L18: Balanced & AVL Trees

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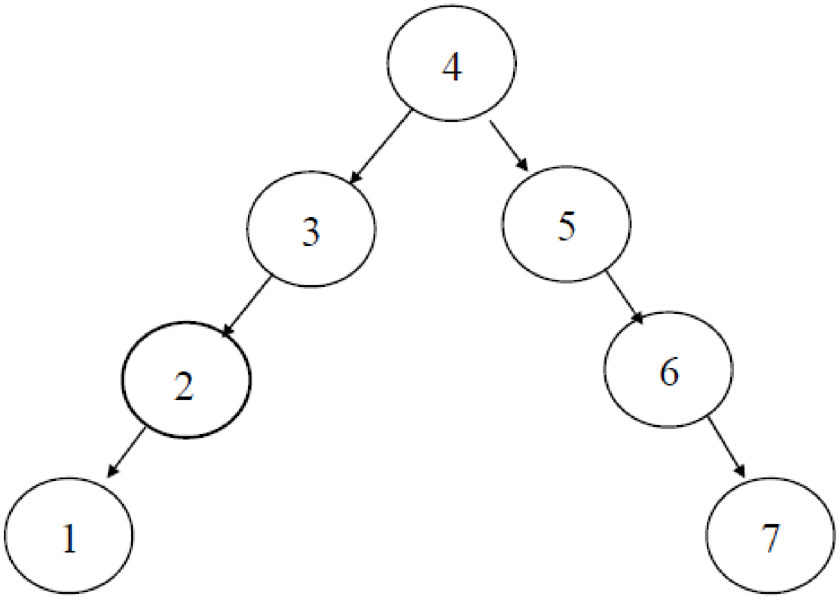
Slides courtesy Venkatesh Babu, CDS

Need for Balancing

- BST have search, insert, delete complexity of O(h)
- But h=n in worst case, i.e., operations are O(n)
- Balancing scheme to limit the height of the tree
- `Balance Factor = |height(left subtree) – height(right subtree)|`

Approaches

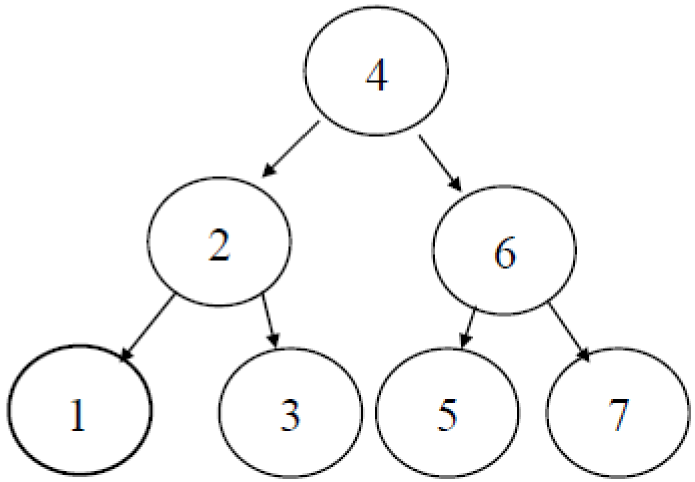
- Balance Factor of root is 0 or 1



Rule #1: Require that the left and right subtrees of the root node have the same height. We can do better.

Approaches

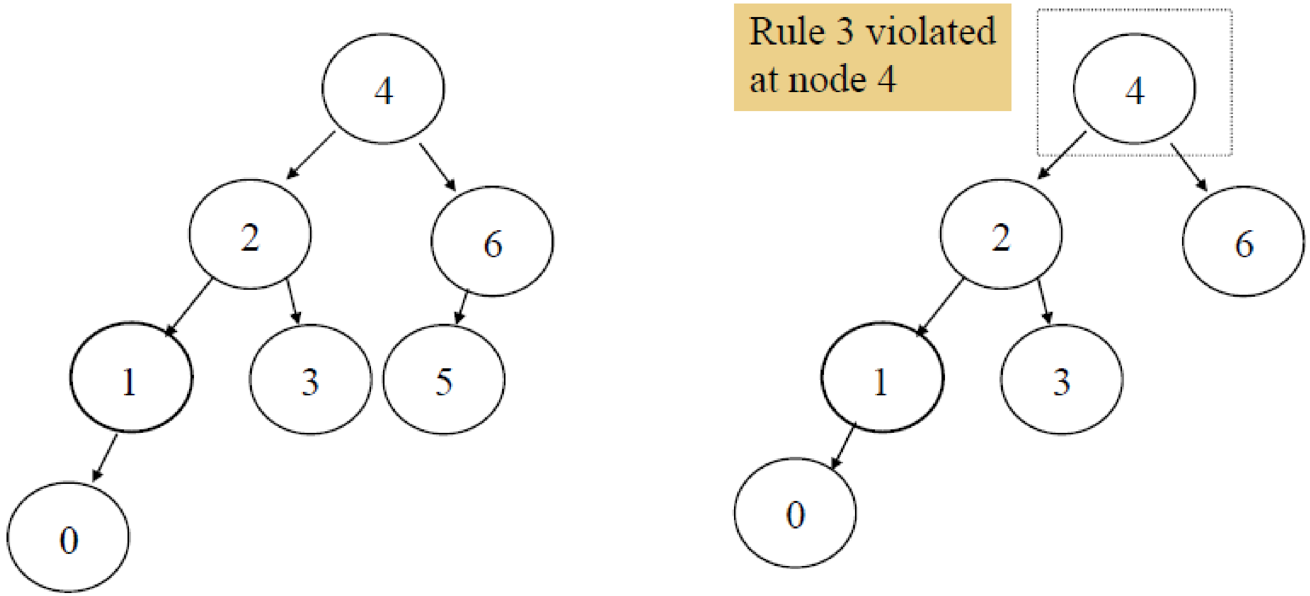
- Balance factor at every node is 0



Rule #2: Require that every node have left and right subtrees of the same height. Too restrictive.

Approaches

- Balance factor at every node is 0 or 1 … **AVL Tree**

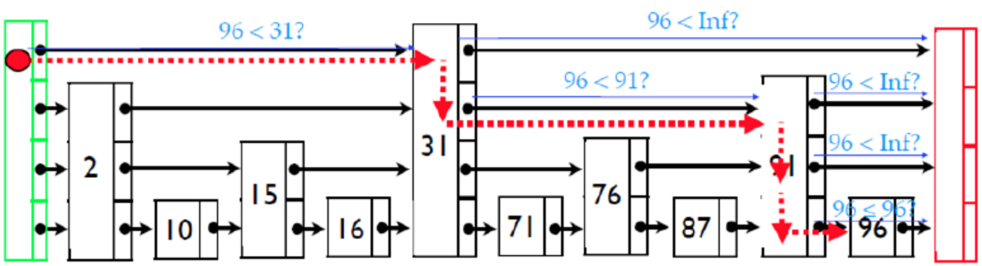


Rule #3: Require that, for every node, the height of the left and right subtrees can differ by most one. The example on the left satisfies rule #3, while the one on the right does not. Why? This rule is a nice compromise between too lax and too restrictive.

Sep 10, 2010 7

Approaches

- Skip Lists (!)
  - Probabilistic, expect same # of nodes in each level
  - Simpler than tree balancing



- "Self Balancing Trees"
- Red-Black Tree
  - Nodes colored red or black. Root is black.
  - Red node must have black children.
  - Number of black nodes from root to leaf is same
- AVL Trees (today)
- B-Trees (next class)

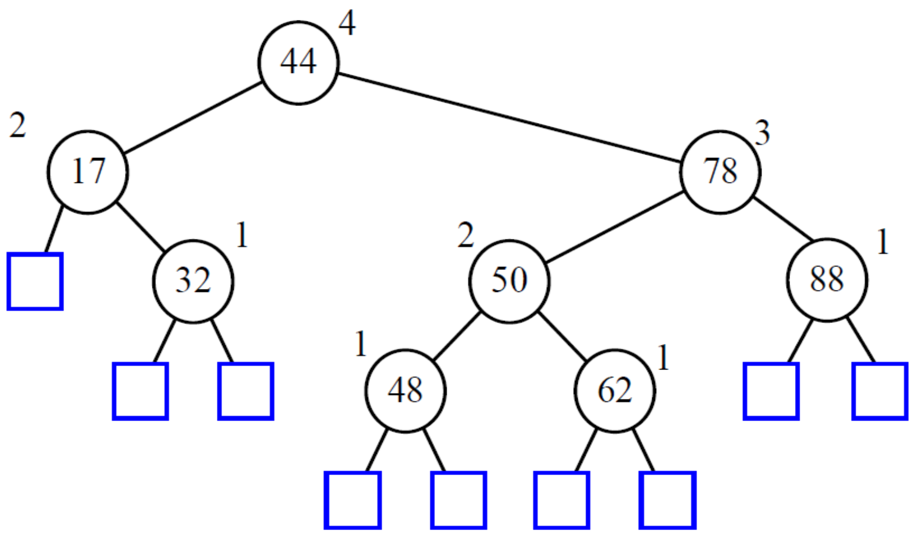
AVL Trees

- Height Balanced Tree
- Georgy **A**delson-**V**elsky and Evgenii **L**andis
  - Soviet Scientists
  - "An algorithm for the organization of information“, 1962
- O(log(n)) insert, delete, search complexity

https://en.wikipedia.org/wiki/AVL_tree

Height Balancing

- Balance Factor = |height(left subtree) – height(right subtree)|
- AVT trees have a balance factor of 1
- Store the heights of subtrees at each level



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Height of an AVL Tree

- $h=O(\log n)$ for an AVL tree with n nodes
- $N(h)$ is the minimum number of nodes in an AVL tree of height h

$$N(1) = 1, N(2) = 2$$

- Since the heights of a node's children differ at most by 1

$$N(h) = 1 + N(h-1) + N(h-2)$$

- Since $N(h-1) > N(h-2)$

$$N(h) > 2.N(h-2)$$

$$N(h) > 4.N(h-4)$$

$$N(h) > 2^i.N(h-2i)$$

$$N(h) > 2^{h/2}$$

$$h<2.\log(N(h))$$

- So, h = O(log(n))

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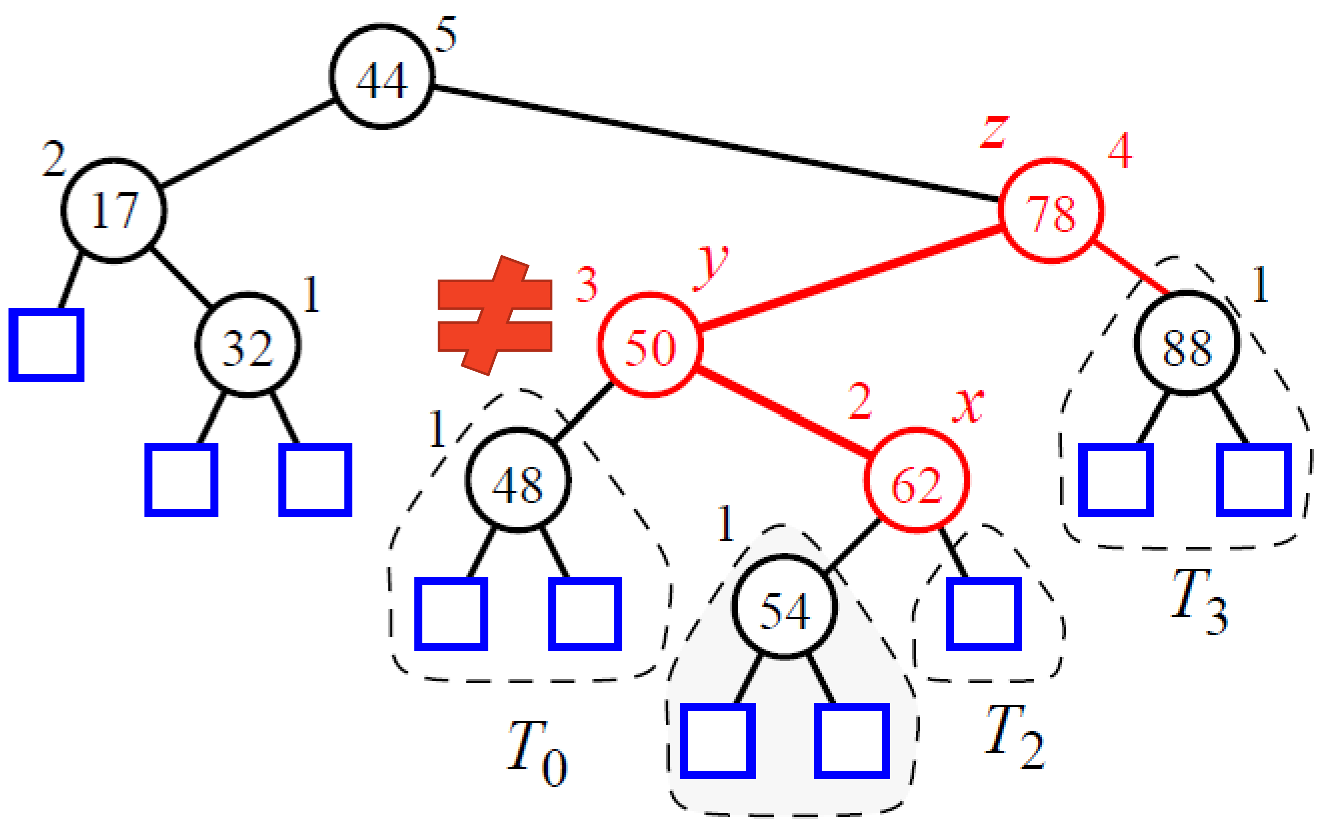
Repairing AVL Trees

- Insertions or deletions can cause the height balance property to be violated
  - Balance factor may become 2
  - *Not more than that for a single insert/delete*
- Repair the tree when imbalance is encountered
  - Go up the tree to parent or grandparent
  - “Rotate the nodes” to restore balanced property

Repairing AVL Trees



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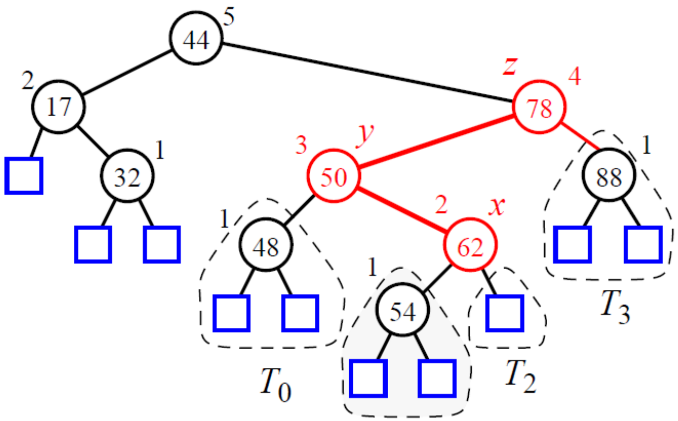
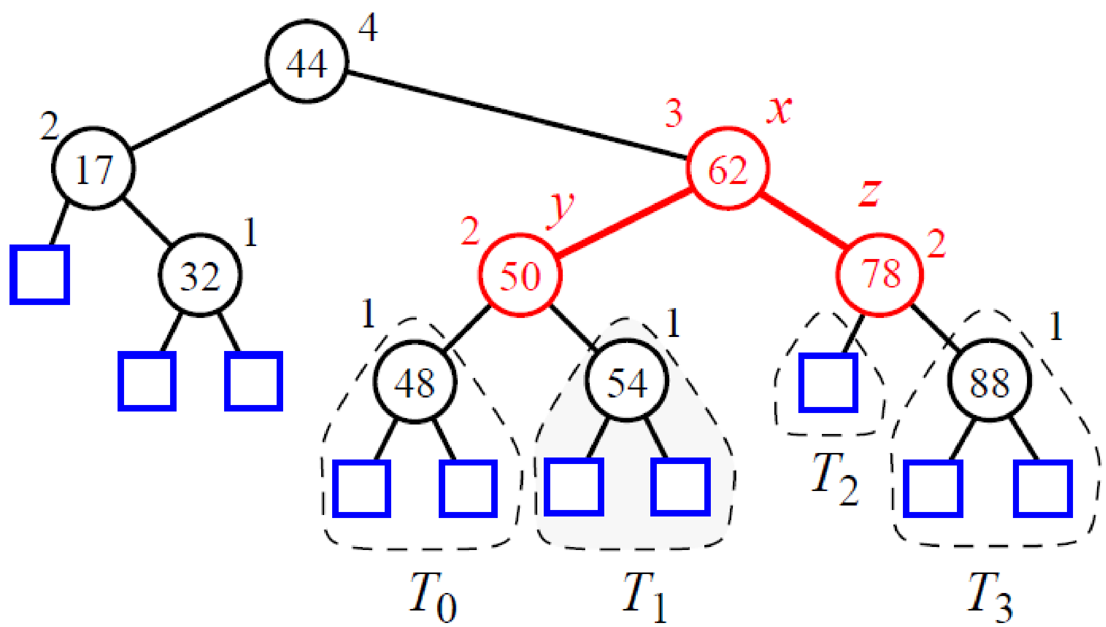
Repairing AVL Trees





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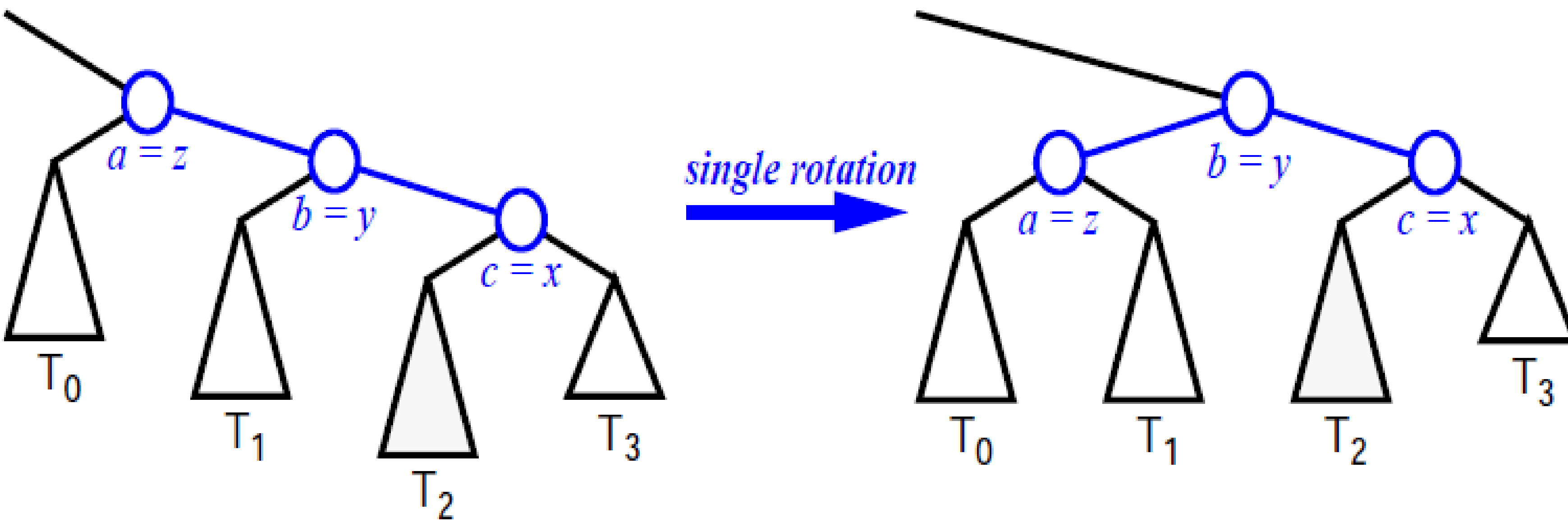
AVL Rotations: Outside Case, Single Rotation



- After single rotation, the new height of the entire subtree is exactly the same as the height of the original subtree prior to the insertion of the new data item

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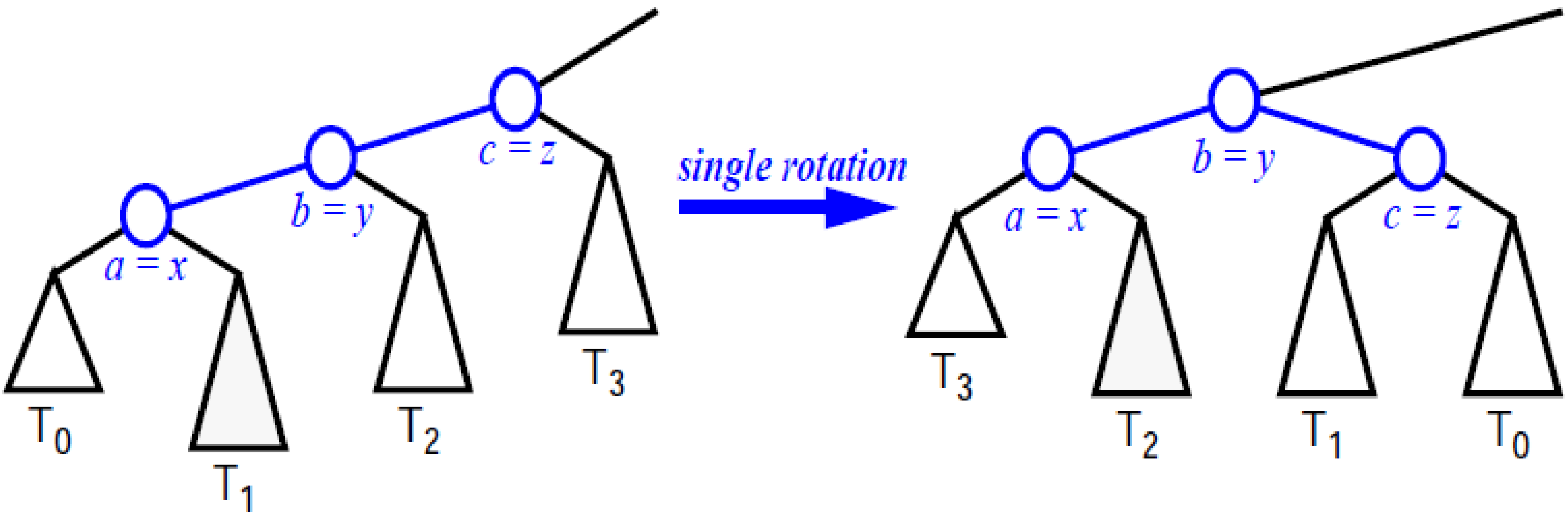
SE-286: Data Structures, Virendra Singh, IISc, 2010

AVL Rotations: Outside Case, Single Rotation



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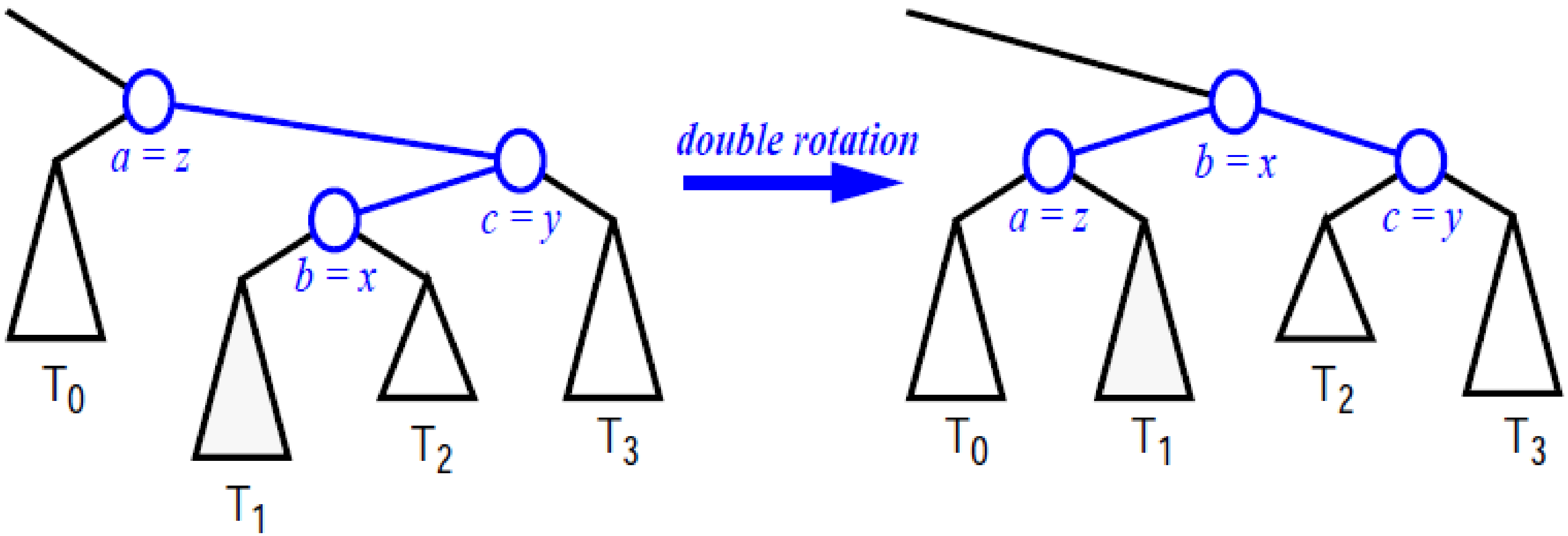
AVL Rotations: Inside Case, Double Rotation



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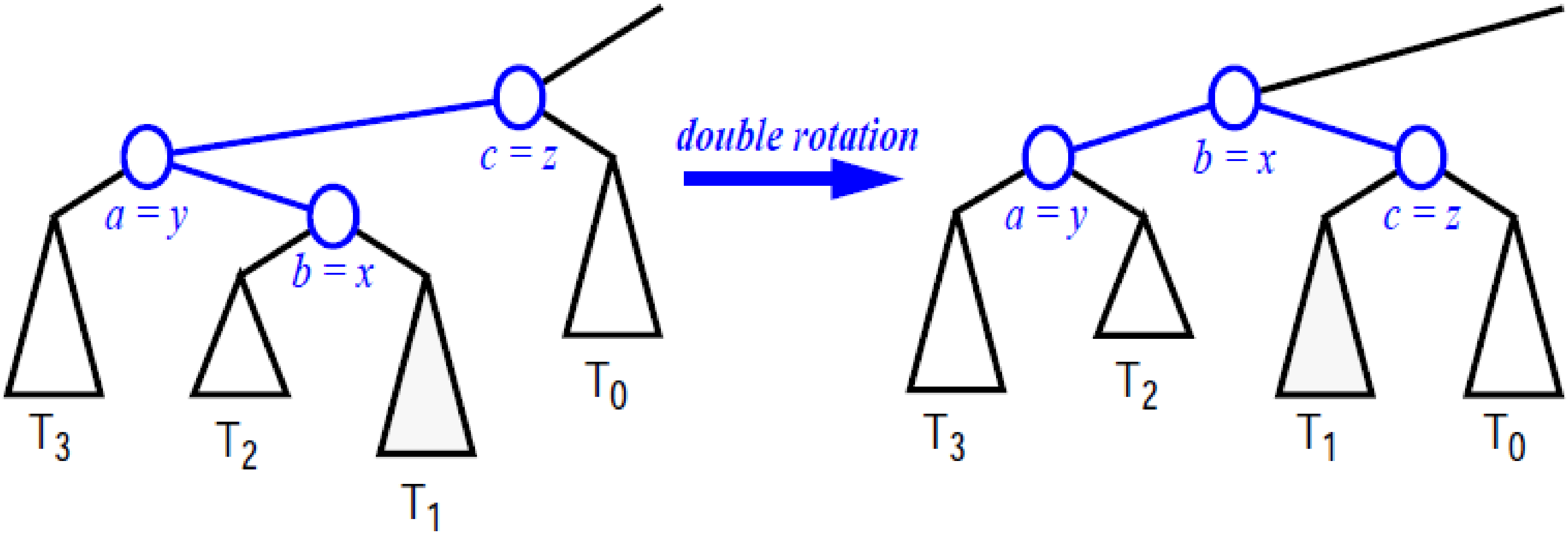
AVL Rotations: Inside Case, Double Rotation



As with the single rotations, double rotations restore the height of the subtree to what it was before the insertion.

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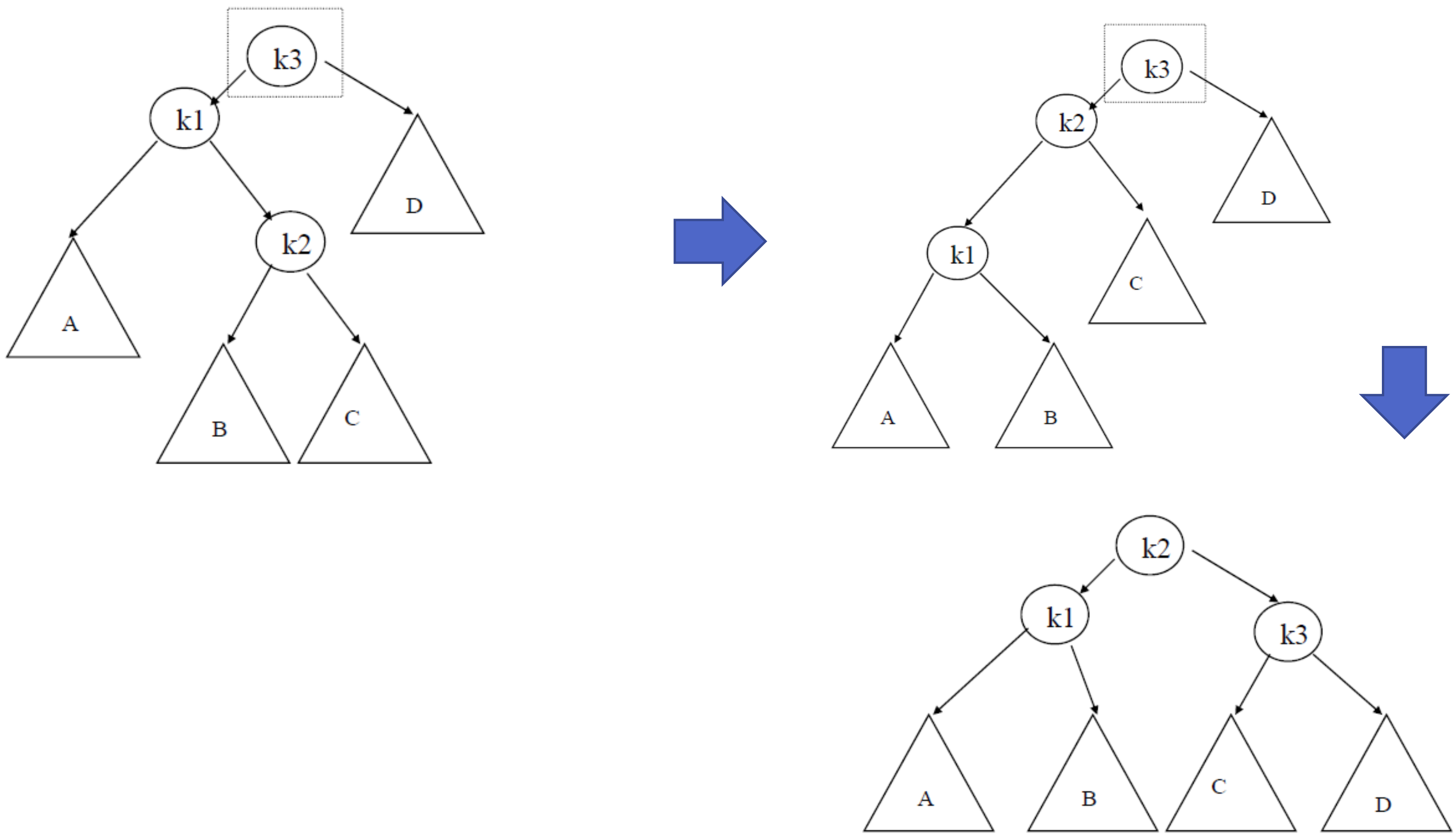
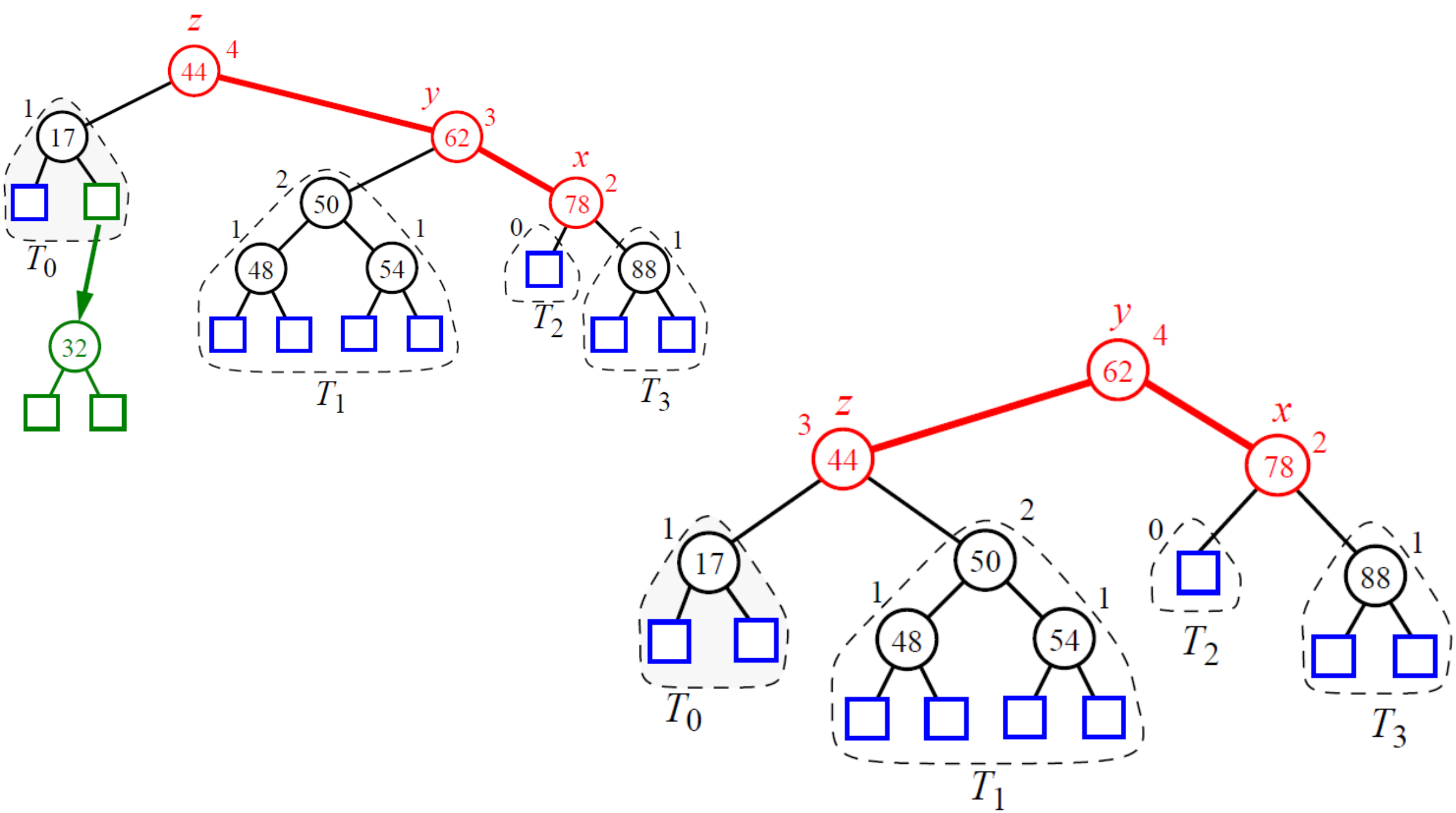
AVL Rotations:
Double Rotation = 2 Single Rotations



AVL Rotations: Deletion



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Tasks

- Self study
  - **Read**: AVL Rotations (online sources)
- Finish Assignment 4 by Wed Oct 26 *(75 points)*
- Make progress on CodeChef (100 points)

Questions?