PRAM Algorithms

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PRAM Model - Introduction

• Parallel Random Access Machine
• Helps to write precursor parallel algorithm without any architecture constraints
• Allows parallel-algorithm designers to treat processing power as unlimited
• Ignores complexity of inter-process communication
Benefits of PRAM

• Can be a suitable basis for the design of a parallel program targeted to a real machine
• Base algorithm can help establish tight lower and upper complexity bounds for practical implementations
• Assumptions made in PRAM model for ideal parallelism can help architecture designers to develop innovative designs
• Can be suitable for modern day architectures, e.g., GPUs
PRAM Architecture Model

• Consists of control unit, global memory, and an unbounded set of processors, each with own private memory
• An active processor reads from global memory, performs computation, writes to global memory
• Execute in SIMD model
Different Models

• Various PRAM models differ in how they handle read or write conflicts

1. EREW - Exclusive Read Exclusive Write
2. CREW - Concurrent Read Exclusive Write
3. CRCW
   1. COMMON - All processors writing to same global memory must write the same value
   2. ARBITRARY - one of the competing processor’s value is arbitrarily chosen
   3. PRIORITY - processor with the lowest index writes its value
Mapping Between Models

• Any PRAM model/algorithm can execute any other PRAM model/algorithm

• For example, possible to convert PRIORITY PRAM to EREW PRAM
Steps in PRAM Algorithm & Example: Reduction

- PRAM algorithms have two phases:
  - Phase 1: Sufficient number of processors are activated
  - Phase 2: Activated processors perform the computations in parallel
- For example, binary tree reduction can be implemented using n/2 processors
- EREW PRAM suffices for reduction
Example: Prefix Sum Calculations

- \[ B[i] = \sum_{j=0}^{i} A[j] \]

- Can be used for separating an array into two categories, lock-free synchronization in shared memory architectures etc.

- CREW PRAM algorithm for prefix sum calculations.

- Can use n/2 processors. Takes O(log \( n \)) time
CREW PRAM for Prefix Sum

- Distance between the elements that are summed are doubled in every iteration
Example: Merging Two Sorted Lists

• Most PRAM algorithms achieve low time complexity by performing more operations than an optimal RAM algorithm

• For example, a RAM algorithm requires at most \( n - 1 \) comparisons to merge two sorted lists of \( n/2 \) elements. Time complexity is \( O(n) \)

• CREW PRAM algorithm:
  • Assign each list element its own processor - \( n \) processors
Example: Merging Two Sorted Lists

• The processor knows the index of the element in its own list
• Finds the index in the other list using binary search
• Adds the two indices to obtain the final position
• The total number of operations had increased to $O(n \log n)$
• Not cost-optimal
Example: Enumeration sort

• Computes the final position of each element by comparing it with the other elements and counting the number of elements having smaller value
• A special CRCW PRAM can perform the sort in $O(1)$ time
• Spawn $n^2$ processors corresponding to $n^2$ comparisons
• Special CRCW PRAM – If multiple processors simultaneously write values to a single memory location, the sum of the values is assigned to that location
Example: Enumeration sort

- So, each processor compares $a[i]$ and $a[j]$. If $a[i] > a[j]$, writes position[i] = 1, else writes position[i] = 0
- So the summation of all positions will give the final position – constant time algorithm
- But not cost-optimal – takes $O(n^2)$ comparisons, but a sequential algorithm does $O(n\log n)$ comparisons
Summary

• PRAM algorithms mostly theoretical
• But can be used as a basis for developing efficient parallel algorithm for practical machines
• Can also motivate building specialized machines