Prefix Computations

Parallel Algorithm: Prefix computations on arrays

- Array X partitioned into subarrays
- Local prefix sums of each subarray calculated in parallel
- Prefix sums of last elements of each subarray written to a separate array Y
- Prefix sums of elements in Y are calculated.
- Each prefix sum of Y is added to corresponding block of X
- Divide and conquer strategy

Example

123456789

Divide

123 456 789

Local prefix sum

1,3,6 4,9,157,15,24

6,15,24

Passing last elements to a processor

6,21,45

Computing prefix sum of last elements on the processor

1,3,6,10,15,21,28,36,45

Adding global prefix sum to local prefix sums in each processor

Prefix sum (SCAN) for shared memory and using CUDA

Example: Scan or Parallel-prefix sum

Definition: *The* all-prefix-sums *operation takes a binary associative operator* $\oplus$*, and an array of* n *elements*

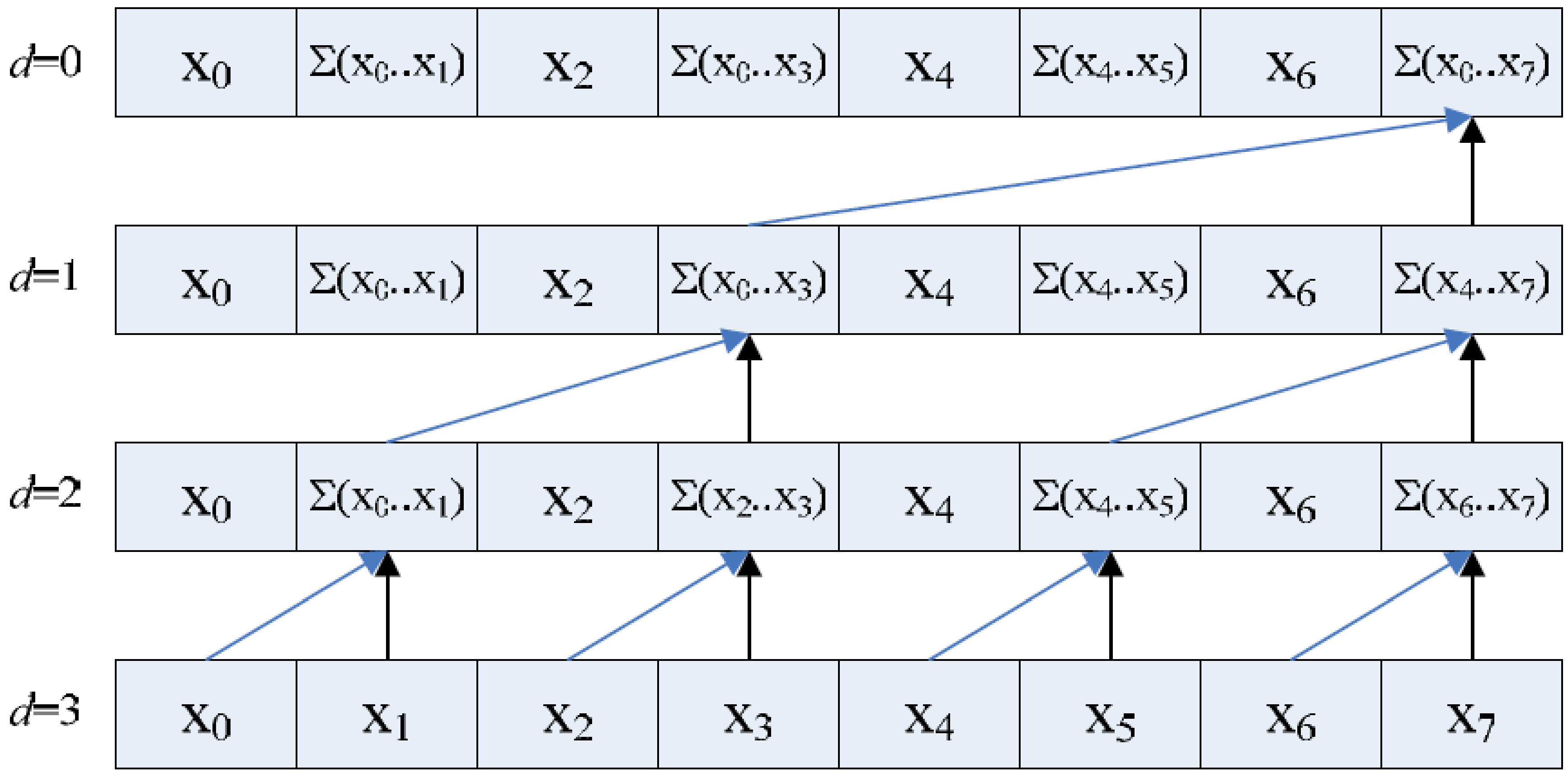
$$[a_0, a_1, \ldots, a_{n-1}],$$

and returns the array

$$[a_0, (a_0 \oplus a_1), \ldots, (a_0 \oplus a_1 \oplus \ldots \oplus a_{n-1})].$$

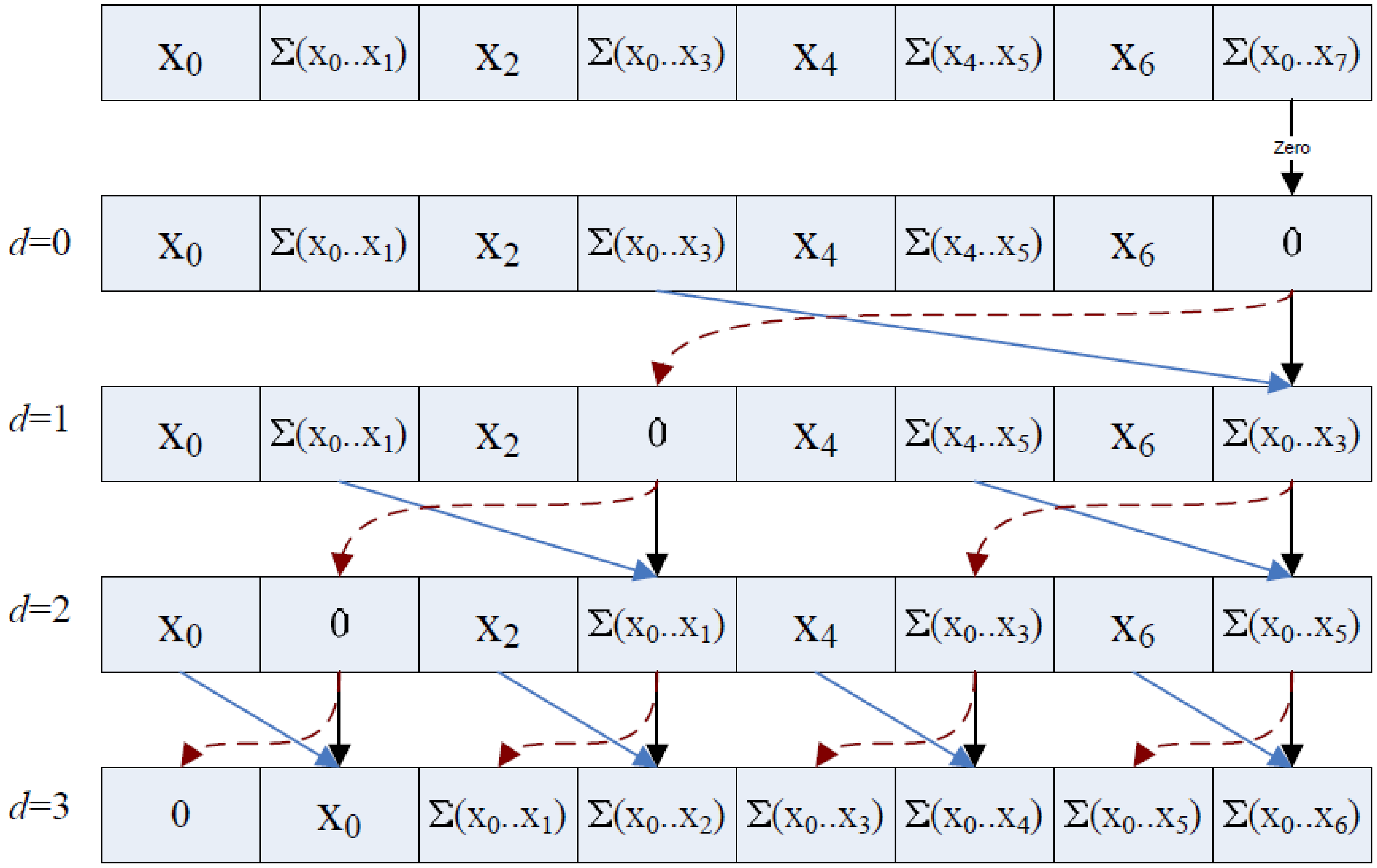
- Using binary tree
- An upward reduction phase (reduce phase or up-sweep phase)
  - Traversing tree from leaves to root forming partial sums at internal nodes
- Down-sweep phase
  - Traversing from root to leaves using partial sums computed in reduction phase

Up Sweep



d=0
x0
Σ(x0..x1)
x2
Σ(x0..x3)
x4
Σ(x4..x5)
x6
Σ(x0..x7)
d=1
x0
Σ(x0..x1)
x2
Σ(x0..x3)
x4
Σ(x4..x5)
x6
Σ(x4..x7)
d=2
x0
Σ(x0..x1)
x2
Σ(x2..x3)
x4
Σ(x4..x5)
x6
Σ(x6..x7)
d=3
x0
x1
x2
x3
x4
x5
x6
x7

Down Sweep



x0
Σ(x0..x1)
x2
Σ(x0..x3)
x4
Σ(x4..x5)
x6
Σ(x0..x7)
Zero
d=0
x0
Σ(x0..x1)
x2
Σ(x0..x3)
x4
Σ(x4..x5)
x6
0
d=1
x0
Σ(x0..x1)
x2
0
x4
Σ(x4..x5)
x6
Σ(x0..x3)
d=2
x0
0
x2
Σ(x0..x1)
x4
Σ(x0..x3)
x6
Σ(x0..x5)
d=3
0
x0
Σ(x0..x1)
Σ(x0..x2)
Σ(x0..x3)
Σ(x0..x4)
Σ(x0..x5)
Σ(x0..x6)

Host Code

- int main(){
-     const unsigned int num_threads = num_elements / 2;
-     /* cudaMalloc d_idata and d_odata */
-     cudaMemcpy(d_idata, h_data, mem_size,   cudaMemcpyHostToDevice));

-      dim3 grid(256, 1, 1); dim3 threads(num_threads, 1, 1);
-      scan<<< grid, threads>>> (d_odata, d_idata);

-      cudaMemcpy(h_data, d_odata[i], sizeof(float) * num_elements, cudaMemcpyDeviceToHost
-      /* cudaFree d_idata and d_odata */
- }

Device Code

```
__global__ void scan_workefficient(float *g_odata, float *g_idata, int n)
{
    // Dynamically allocated shared memory for scan kernels
    extern  __shared__  float temp[];

    int thid = threadIdx.x;    int offset = 1;

    // Cache the computational window in shared memory
    temp[2*thid]   = g_idata[2*thid];
    temp[2*thid+1] = g_idata[2*thid+1];
```

Device Code

```
// build the sum in place up the tree
for (int d = n>>1; d > 0; d >>= 1)
{
    __syncthreads();

    if (thid < d)
    {
        int ai = offset*(2*thid+1)-1;
        int bi = offset*(2*thid+2)-1;

        temp[bi] += temp[ai];
    }

    offset *= 2;
}
```

Device Code

```
// scan back down the tree

// clear the last element
if (thid == 0)       temp[n - 1] = 0;
// traverse down the tree building the scan in place
for (int d = 1; d < n; d *= 2)
{
    offset >>= 1;
    __syncthreads();
    if (thid < d)
    {
        int ai = offset*(2*thid+1)-1;
        int bi = offset*(2*thid+2)-1;
```

Device Code

```
            float t   = temp[ai];
            temp[ai]  = temp[bi];
            temp[bi] += t;
        }
    }

    __syncthreads();

    // write results to global memory
    g_odata[2*thid]   = temp[2*thid];    g_odata[2*thid+1] = temp[2*thid+1];
}
```

Parallel Sorting

Sathish Vadhiyar

Parallel Sorting Problem

- The input sequence of size N is distributed across P processors

- The output is such that
  - elements in each processor P_i is sorted
  - elements in P_i is greater than elements in P_{i-1} and lesser than elements in P_{i+1}

Parallel quick sort

- Naïve approach

- Start with a single processor; divide array into two sub-arrays
- Now involve one more processor
- Both the processors perform the next step of quick sort within their local subarrays
- And so on….till the number of subarrays equal the number of processors

- Disadvantage: Inefficient utilization of processors

Another algorithm

- This algorithm involves all the processors in all the iterations
- One of the processors, P0, begins by broadcasting one of its elements as the pivot element to all the processors
- Each processor then divides its local array into two sub-arrays
  - L_i: elements less than the pivot
  - G_i: elements greater than the pivot

Parallel Quick Sort

- Processors then divided into two groups:
  - First group will process the subsequent steps with L_i s
  - Second group with G_i s
- The sizes of the processor groups must be in the ratio of the number of elements in Ls and Gs to achieve load balance
- These number of elements can be found using an allreduce operation

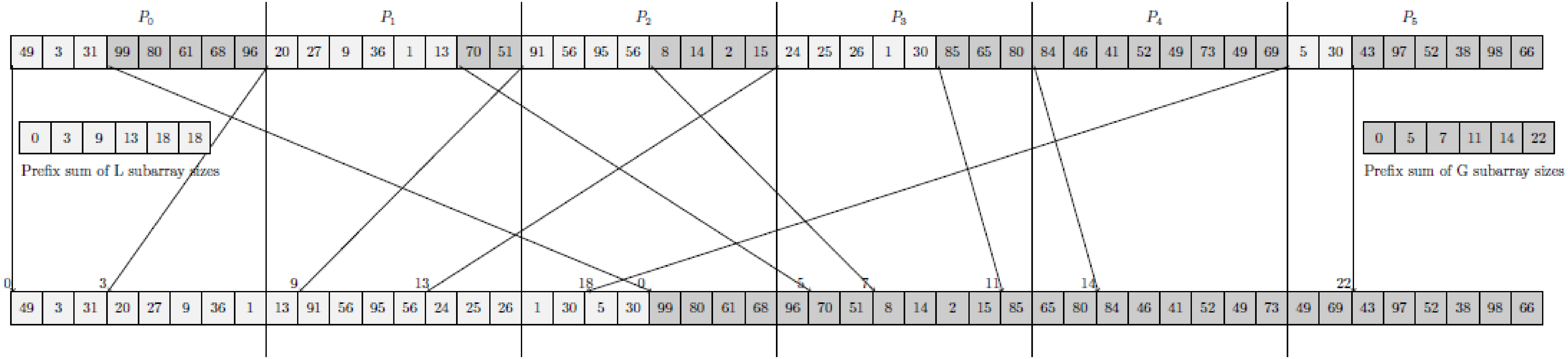
Shared memory implementation

- All L's are formed in the first part of the array; all G's in the second part
- Each processor needs to know the locations in the shared memory where it has to write its L_i and G_i
- Prefix sums of the sizes of the subarrays can be used

- Prefix sum can be done in O(logP)

Example: Prefix sum illustration

- In this example, 36 is the pivot element



Message Passing Version

- A processor should know which elements in its Li and Gi it should send to which processor
- Distributed prefix sum is used
- A processor can then deduce its destination processor for sending its L array using:
  - Total number of elements of L subarrays
  - prefix sums of sizes
  - Size of the processor group that will be responsible for L subarray
- Similarly for the G subarray
- In worst case, this requires all-to-all with time complexity O(N/P)

Parallel Quick sort

- The process now repeats with the subgroups
- Until the number of subgroups equal the number of processors
- At this stage, each processor performs a local quick sort: $O(N/P\log(N/P))$

Complexity and analysis

- log P times:
  - Broadcast: O(logP)
  - Allreduce: O(logP)
  - Prefix sum and all-to-all: O(logP + N/P)
- Then local quick sort: O(N/P.logP)

- Total: O(N/P.logP) + O($\log^2$P+N/P.logP)

- Weaknesses: Load imbalance and under-utilization

Graph Algorithms

Graph Algorithms

Sathish Vadhiyar

Graph Traversal

- Graph search plays an important role in analyzing large data sets
- Relationship between data objects represented in the form of graphs
- Breadth first search used in finding shortest path or sets of paths

Parallel BFS
Level-synchronized algorithm

- Proceeds level-by-level starting with the source vertex
- Level of a vertex – its graph distance from the source
- Also, called **frontier-based** algorithm
- The parallel processes process a level, synchronize at the end of the level, before moving to the next level – Bulk Synchronous Parallelism (**BSP**) model
- How to decompose the graph (vertices, edges and adjacency matrix) among processors?

Distributed BFS with 1D Partitioning

- Each vertex and edges emanating from it are owned by one processor
- 1-D partitioning of the adjacency matrix

$$\left[\begin{array}{c} A_1 \\ \hline A_2 \\ \hline \vdots \\ \hline A_P \end{array}\right]$$

- Edges emanating from vertex v is its edge list = list of vertex indices in row v of adjacency matrix A

1-D Partitioning

- At each level, each processor owns a set F – set of frontier vertices owned by the processor
- Edge lists of vertices in F are merged to form a set of neighboring vertices, N
- Some vertices of N owned by the same processor, while others owned by other processors
- Messages are sent to those processors to add these vertices to their frontier set for the next level

Algorithm 1 Distributed Breadth-First Expansion with 1D Partitioning

Initialize $L_{v_s}(v) = \begin{cases} 0, & v = v_s, \text{ where } v_s \text{ is a source} \\ \infty, & \text{otherwise} \end{cases}$
for $l = 0$ to ∞ **do**
  $F \leftarrow \{v \mid L_{v_s}(v) = l\}$, the set of local vertices with level l
  if $F = \emptyset$ for all processors **then**
    Terminate main loop
  end if
  $N \leftarrow$ {neighbors of vertices in F (not necessarily local)}
  for all processors q **do**
    $N_q \leftarrow$ {vertices in N owned by processor q}
    Send N_q to processor q
    Receive $\bar{N}_q$ from processor q
  end for
  $\bar{N} \leftarrow \bigcup_q \bar{N}_q$ (The $\bar{N}_q$ may overlap)
  for $v \in \bar{N}$ and $L_{v_s}(v) = \infty$ **do**
    $L_{v_s}(v) \leftarrow l + 1$
  end for
end for

$L_{vs}(v)$ – level of v, i.e, graph distance from source vs

BFS on GPUs

```
bfs_kernel(int curLevel){
v = blockIdx.x * blockDim.x + threadIdx.x;
if dist[v] == curLevel then
    forall the n ∈ neighbors(v) do
        if visited[n] == 0 then
            dist[n] = dist[v] + 1;
            visited[n] = 1;
        end
    end
end
}
```

BFS on GPUs

- One GPU thread for a vertex
- For each level, a GPU kernel is launched with the number of threads equal to the number of vertices in the graph
- Only those vertices whose assigned vertices are frontiers will become active
- Do we need atomics?
- Severe load imbalance among the treads
- Scope for improvement

Thank You