Supplementary Information for

Siamese-SR: A Siamese Super-Resolution model for boosting resolution of Digital Rock images for improved petrophysical property estimation

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The supplementary material is organized as follows:

- Section 1 : Algorithm for training our proposed Siamese-SR model
- Section 2 : Mercury Injection Capillary Pressure (MICP) simulation curves

1 Algorithm for training our proposed Siamese-SR model

Algorithm 1: Training GANs with Siamese Discriminator

Data: epochs : Total number of training epochs, batchs: Total number of batches, $\tilde{G}$: Generator, $\tilde{D}$: Relativistic Discriminator, $\tilde{S}$: Siamese Discriminator, LR: Low Resolution Image, SR: Super-Resolved Image

for *epoch= 1:epochs* **do**

  for *batch = 1:batchs* **do**

    x = LR;

    y = HR;

    /* Generate SR image from LR image */

    $\hat{y} = \tilde{G}(x)$;

    /* Train Relativistic discriminator using HR(real) data */

    train($\tilde{D}$, y);

    /* Train Relativistic discriminator using generated SR(fake) data */

    train($\tilde{D}$, $\hat{y}$);

    /* Calculate amount of perturbation to be added in HR */

    perturbation = Average(x) − Average(y);

    /* Generate perturbed target */

    y* = y + perturbation;

    /* Train Siamese discriminator using (HR, HR*) matching input pair */

    train($\tilde{S}$, (y, y*));

    /* Train Siamese discriminator using (SR, HR) non-matching input pair */

    train($\tilde{S}$, (y, $\hat{y}$));

    /* Freeze the weights of both discriminator */

    $\tilde{D}$.trainable = False;

    $\tilde{S}$.trainable = False;

    /* Train generator using adversarial losses ($L_{Siamese}$ & L_{Re}) and image restoration losses ($\mathcal{L}^{OIR}$) */

    train($\tilde{G}$, x);

    /* Make weights of both discriminator trainable */

    $\tilde{D}$.trainable = True;

    $\tilde{S}$.trainable = True;

Figure S1: Algorithm to train GANs with Siamese Discriminator

2 Mercury Injection Capillary Pressure (MICP) simulation curves

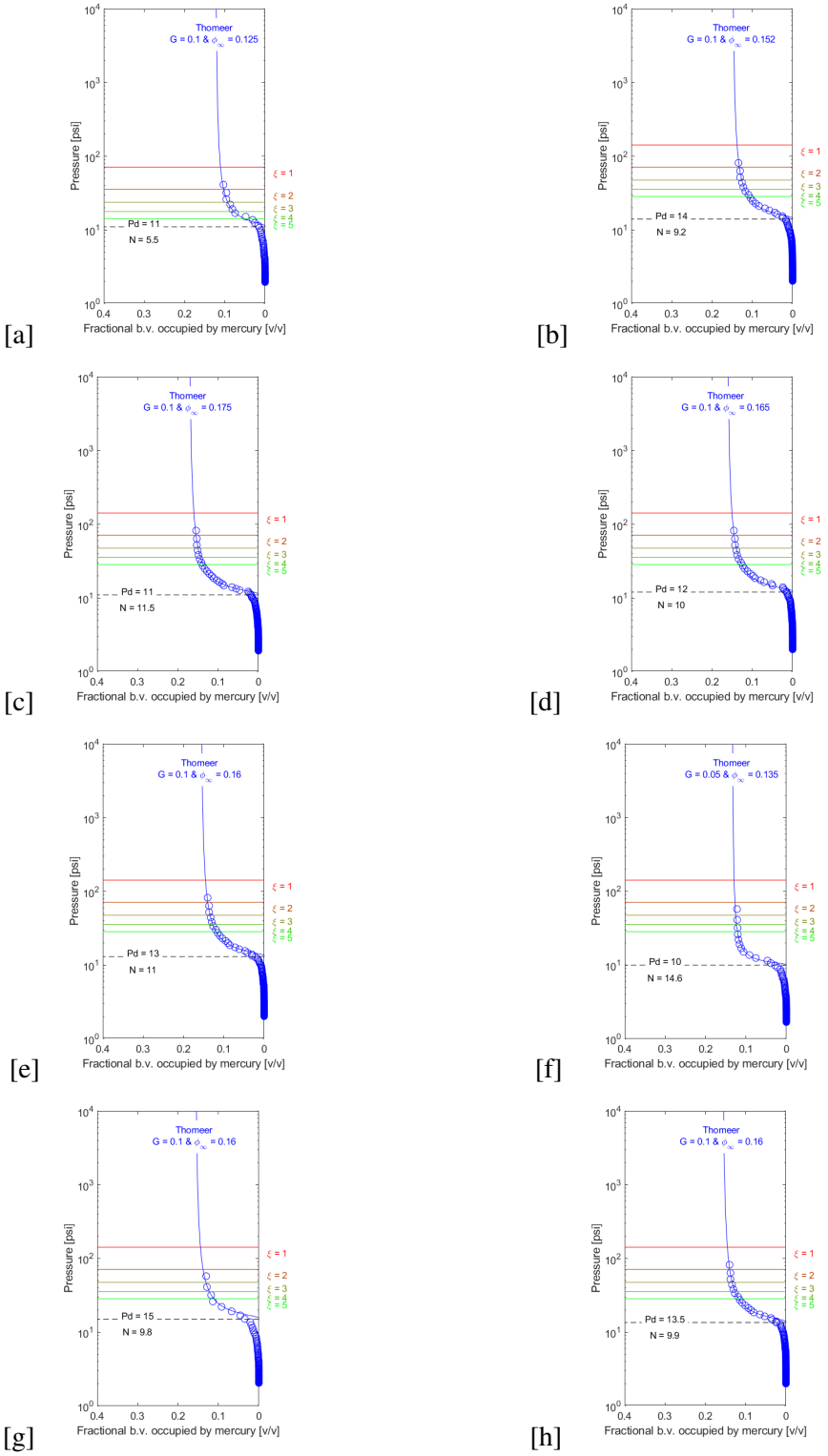

Figure S2: Mercury Injection Capillary Pressure (MICP) simulation curves using isotropic boundary conditions fitted with Thomeer equation to obtain petrophysical properties of interest for (a) LR (b) HR (c) SRGAN (d) ESRGAN (e) ESRGAN-MINC (f) EDSR (g) SPSR and (h) Siamese-SR (Proposed).

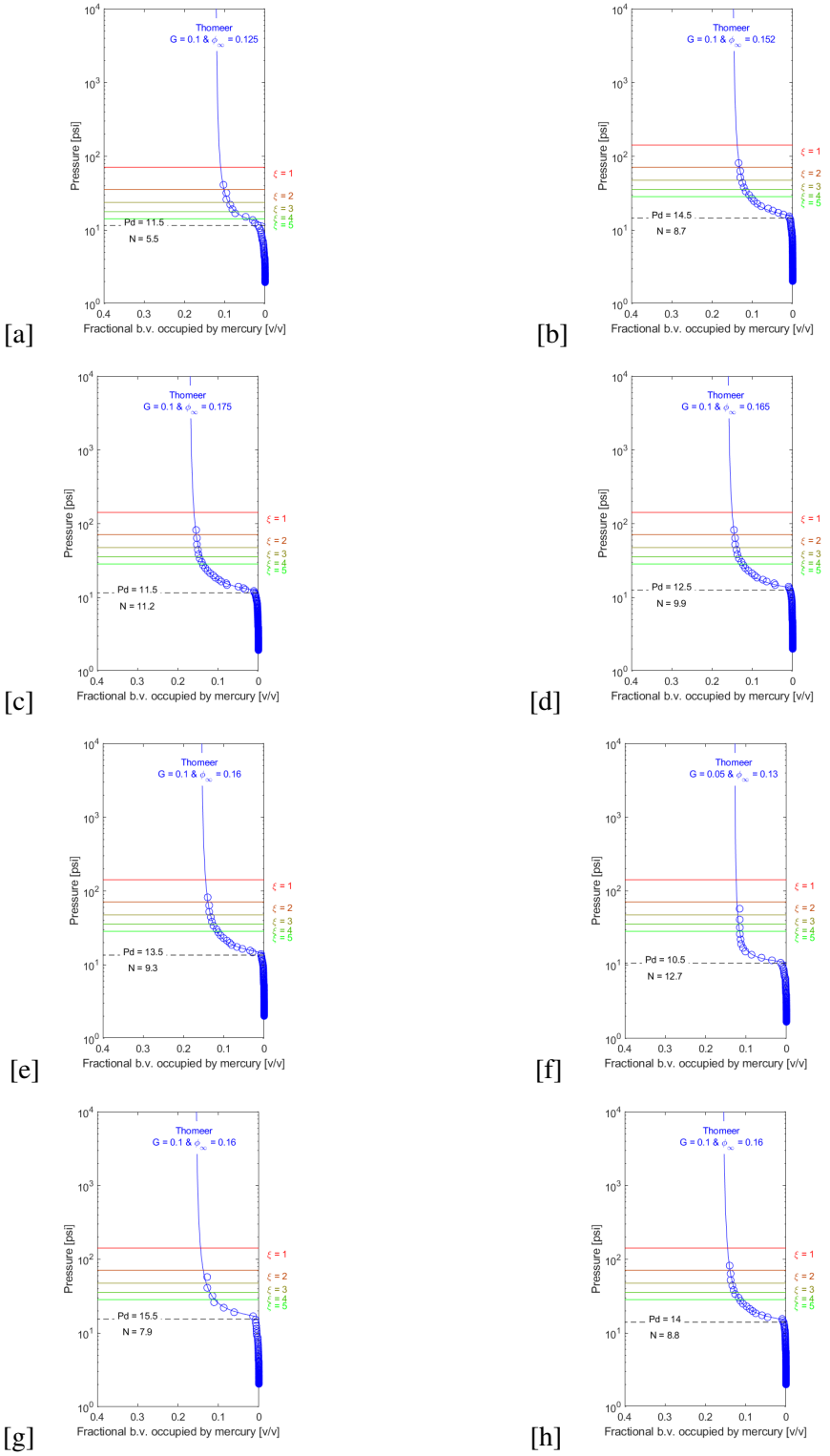

Figure S3: Mercury Injection Capillary Pressure (MICP) simulation curves using anisotropic boundary conditions fitted with Thomeer equation to obtain petrophysical properties of interest for (a) LR (b) HR (c) SRGAN (d) ESRGAN (e) ESRGAN-MINC (f) EDSR (g) SPSR and (h) Siamese-SR (Proposed).