Data-resolution based optimal choice of minimum required measurements for image-guided diffuse optical tomography

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Image-guided diffuse optical tomography has the advantage of reducing the total number of optical parameters being reconstructed to the number of distinct tissue types identified by the traditional imaging modality, converting the optical image-reconstruction problem from underdetermined in nature to overdetermined. In such cases, the minimum required measurements might be far less compared to those of the traditional diffuse optical imaging. An approach to choose these optimally based on a data-resolution matrix is proposed, and it is shown that such a choice does not compromise the reconstruction performance.

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Image-guided diffuse optical tomography has been shown to provide better diagnostic information compared to the information provided by individual modalities separately, where the image guidance refers to the usage of structural information provided by traditional imaging modalities (examples being magnetic resonance imaging [MRI], computed tomography, and ultrasound) [1–3]. Utilization of structural information provided by the traditional imaging modalities to improve the image-reconstruction performance has been investigated extensively [1].

The usage of structural information to reduce the reconstructed number of optical parameters to the number of distinct tissue types determined by the traditional imaging modalities (also known as *hard priors*) is shown to have a computational advantage compared to other image-guided procedures [3–6]. In this approach, the assumption is that the optical property value in each distinct region as identified by the traditional imaging modality is uniform and can have distinct values across regions. This makes the reconstructed image quality dependent only on the prior information, and the reconstruction procedure is about estimation of the optical property value (quantification). Even though the number of optical parameters to be reconstructed is much lower in this approach compared to traditional diffuse optical tomographic imaging [3–6], typically the same number of measurements are utilized, which might be unnecessary. Reduction of the minimum number of required measurements without compromising the quantification of the reconstructed optical properties is highly desirable as it has implications in terms of reduced time to complete the data-acquisition procedure, especially in the dynamic imaging case [5,6].

In this Letter, a novel approach, to the best of our knowledge, that can use the image guidance to reduce the minimum required measurements substantially was attempted. This work was motivated by the recent work on the optimization of the data-collection strategy based on data-resolution characteristics for traditional diffuse optical tomographic imaging [7]. It will be shown through usage of numerical experiments involving a realistic breast tissue case and gelatin phantom experiments that the quantification (contrast recovery) of reconstructed optical parameters using the minimum measurements determined by the proposed method is similar to the results obtained using all measurements. As the emphasis is on optimal choice of minimum required measurements using the proposed method, the discussion is limited to continuous-wave (CW) two-dimensional imaging domains.

The CW near-infrared light propagation in thick, soft biological tissues, such as breast and brain tissue, can be modeled using a diffusion equation [8] given by

$$-\nabla.D(r)\nabla\Phi(r) + \mu_a(r)\Phi(r) = Q_o(r), \tag{1}$$

where $Q_o(r)$ is the isotropic CW source located at position r and $\Phi(r)$ is the photon density (real value). The optical absorption coefficient, which is to be estimated, is given by $\mu_a(r)$. The diffusion coefficient, assumed to be known and constant throughout the imaging domain in the CW case, is represented by $D(r)$ and is equal to $1/3[\mu_a(r) + \mu_s'(r)]$, with $\mu_s'(r)$ representing the reduced scattering coefficient. Due to the versatility provided by the finite-element method (FEM) in terms of handling the irregular geometries, Eq. (1) is solved numerically using FEM [8]. A Robin-type (Type-III) boundary condition is deployed in this numerical scheme to take care of the refractive-index mismatch at the boundary of the imaging domain [8]. The modeled measurements, represented by $G(\mu_a)$, are the sampled values of $\Phi(r)$ for given source-detector (measurement) locations at the boundary of the imaging domain.

The objective function to be minimized with respect to μ_a for the reconstruction problem in image-guided diffuse optical tomography is given by

$$\Omega = \|y - G(\mu_a)\|^2, \tag{2}$$

where y represents the experimentally measured intensity data having dimension of $\text{NM} \times 1$, with NM representing the number of measurements. Note that the dimension of μ_a to be estimated is $\text{NR} \times 1$, with

NR representing the number of distinct regions identified by the traditional imaging modalities. For breast tissue, typically $\mathrm{NR} = 3$, where the segmented regions are fatty, fibroglandular, and tumorous. Typically the optimization is achieved the Lenverg–Marquardt method [3] with the equation being

$$\Delta\mu_a = [\mathbf{J}^T\mathbf{J} + \lambda\mathbf{I}]^{-1}\mathbf{J}^T\delta, \tag{3}$$

where $\mathbf{J}$ represents the Jacobian (also known as the sensitivity matrix), given by $dG(\mu_a)/d\mu_a$, having dimensions of $\mathrm{NM} \times \mathrm{NR}$ and computed using either a perturbation or adjoint method combined with a region mapper [3,8] with $\mathrm{NM} \gg \mathrm{NR}$. The data–model misfit is represented by δ and is equal to $y - G(\mu_a)$. The regularization parameter is represented by λ, which allows the inversion of the ill-conditioned matrix $\mathbf{J}^T\mathbf{J}$, typically reduced by a small factor over the iterations (here it is $10^{0.25}$). The identity matrix is represented by $\mathbf{I}$. The estimation of μ_a is an iterative procedure in which every $\Delta\mu_a$ obtained using Eq. (3) is added to the current μ_a and both $\mathbf{J}$ and δ are recomputed using the updated μ_a. This iterative procedure is stopped when the change in the L2 norm of δ between the successive iterations falls below 2% [3,7].

Taylor expansion of the modeled data $G(\mu_a)$ around an initial guess μ_{a0} and considering up to first-order (linear) terms in combination with assuming a perfect model leads to the well-known equality $\mathbf{J}\Delta\mu_a = \delta'$ (Eq. 9 in [7]), where δ' represents the data–model misfit using a perfect model ($y = G(\mu_a)$). Using $\Delta\mu_a$ given by Eq. (3) in this Letter leads to the definition of the data-resolution matrix as

$$\mathbf{N} = \mathbf{J}[\mathbf{J}^T\mathbf{J} + \lambda\mathbf{I}]^{-1}\mathbf{J}^T, \tag{4}$$

which has dimensions of $\mathrm{NM} \times \mathrm{NM}$ and represents the relation between the perfect model and its linearized version. In the ideal scenario, $\delta = \delta'$, leading to $\mathbf{N} = \mathbf{I}$, which is only possible when $\lambda = 0$. In practice, $\lambda > 0$ due to the ill-conditioned nature of the problem, making the recovered μ_a not equal to the expected μ_a. The closer $\mathbf{N}$ is to $\mathbf{I}$, the smaller are the prediction errors for δ. The magnitude of diagonal entries of $\mathbf{N}$ also indicate the importance of the corresponding data point to its own prediction [7]. The closer the magnitude of diagonal entries to 1, the higher the importance. Note that determination of $\mathbf{N}$ does not depend on δ (or the noise in the data) and is purely characteristic of the underlying model, which includes the prior information used for calculation of $\mathbf{J}$ and the regularization scheme [7].

For determining the optimal minimal measurements, initially $\mathbf{N}$ is computed at the first iteration. Next, the diagonal entries of $\mathbf{N}$ are sorted in the descending order; this sorted list signifies the relative importance of a particular measurement, with the first entry being the most important and the last entry having the least importance. This leads us to the optimal choice of minimum required measurements (M) based on the sorted diagonal entries of $\mathbf{N}$. The reduced Jacobian ($\tilde{\mathbf{J}}$) can be defined as

$$\tilde{\mathbf{J}} = \mathbf{J}(Ind, :), \tag{5}$$

where Ind represents the indices of unsorted diagonal entries of $\mathbf{N}$ corresponding to the first M entries, leading to the dimensionality of $\tilde{\mathbf{J}}$ being $M \times \mathrm{NR}$ ($M \ll \mathrm{NM}$). For example, for breast imaging, ideally (in the noiseless case) $M = 3$ as $\mathrm{NR} = 3$. Real measurements are always corrupted with the noise, leading to $\mathrm{NR} \le M \le \mathrm{NM}$. Determining the exact M for a given estimation problem depends mainly on the condition number of $\tilde{\mathbf{J}}$, with optimal M choice resulting in the condition number of $\tilde{\mathbf{J}}$ being in the same numerical range as the condition number of $\mathbf{J}$. The determination of M was performed at the first iteration and was reused in subsequent iterations.

Numerical experiments using synthetic data generated on a breast MRI image segmented by an FE mesh with 5199 nodes corresponding to 10,208 linear triangular elements is used. The target μ_a distribution is shown in Fig. 1 (top-left corner) consisting of three regions having μ_a values of 0.01 (fatty), 0.015 (fibroglandular), and 0.02 (tumorous) mm^{-1}. The fatty region μ_a value was used as an initial guess for the iterative reconstruction procedure in all cases, which could be easily obtained using a data-calibration procedure [9]. Note that the normalized value of $\mathbf{N}$ does not depend on the μ_a used as long as it is within the acceptable range of normal tissue values (0.005–0.03 mm^{-1}); hence the initial guess's effect on choosing optimal M is negligible. The μ_s' for all three regions was set at 1 mm^{-1} and assumed to be known. Sixteen fibers were arranged in a circular fashion at the indentations of the boundary [given below the text of All (1%) in Fig. 1], where when one fiber acts as source, the remaining 15 act as detectors, resulting in a total of 240 (NM) measurements. The numerically generated data was combined with normally distributed Gaussian noise levels of 1% (close to the expected noise level in a typical experiment) and 10% (extreme case) to test

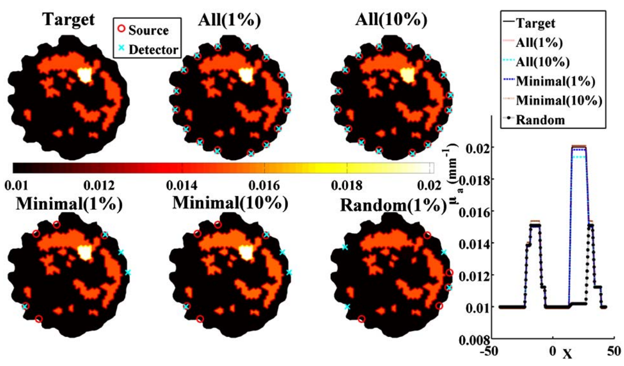

Fig. 1. (Color online) Reconstructed μ_a distributions using all and an optimally chosen minimal number of measurements ($M = 6$ in this case) with numerically generated 1% and 10% noisy data in a realistic breast case (top left, obtained from volunteer). The text on top of each distribution represents the number of measurements used, with the minimum being obtained using the proposed method, where the data noise level is given in parenthesis ($\lambda = 1.5$ for all cases). The corresponding sources and detectors for each case are also indicated along with their corresponding μ_a distribution. The bottom-right corner μ_a distribution was obtained by randomly choosing six measurements. The one-dimensional cross-sectional profile plot passing through the tumor for all reconstruction cases is given at the right side.

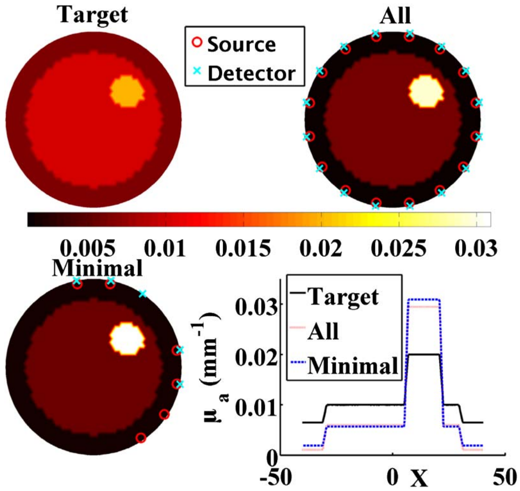

Fig. 2. (Color online) Results of a similar effort to that for Fig. 1 for the case of experimental gelatin-phantom data. Here also the M (minimal measurements) was chosen as 6. The regularization parameter in this case was kept at 0.01.

the robustness of the proposed method. An FE mesh with 2002 nodes corresponding to 3814 elements was used in the reconstruction procedure with μ_a values of the fatty region as the uniform initial guess. The M was chosen initially as 3 and incremented by 1 at each step, the corresponding condition number of $\tilde{\mathbf{J}}$ for each M was computed and compared with the condition number of $\mathbf{J}$. The condition numbers of $\tilde{\mathbf{J}}$ for $M = 3$, 4, 5, and 6 are $4.32e + 03$, $4.19e + 03$, 498.41, and 178.19, respectively, and for $\mathbf{J}$ it is 43.28; hence the optimal M was selected as 6. The reconstructed distributions and one-dimensional profile plots using all (NM = 240) and minimal ($M = 6$) measurements as determined by the proposed method are given in Fig. 1.

The reconstructed μ_a distribution using a random choice of six measurements is also given in Fig. 1 (bottom right). It clearly shows that the optimal choice of minimal measurements (in this case, $M = 6$) did not compromise the reconstructed μ_a quantification and reconstructed results matched with the results obtained using all measurements even for the extreme case of a 10% noise level. When the same number of minimal measurements were chosen at random, results in tumor region μ_a value were equal to the fatty region μ_a value, missing the tumor completely and resulting in a false negative. With a similar effort as in Fig. 1, the results obtained using the experimental data and using the gelatin phantom (height: 25 mm; diameter: 86 mm) that mimics the layered tissue model of the breast [3,9] are given in Fig. 2. The wavelength of the light source used here is 785 nm. The expected (target) μ_a distribution is given in the top-left corner of Fig. 2. The phantom had contrast in μ_s' as well, with values for fatty (outermost region), fibroglandular (middle region), and tumor (smallest circular region) regions being 0.65, 1.0, and 1.2 mm^{-1}, respectively. Here the FE mesh that was used in the reconstruction procedure had 1785 nodes corresponding to 3418 linear triangular elements. The μ_s' was assumed to be known and to have a uniform value of 0.65 mm^{-1}. Here as well, the M value that was found to be optimal was 6. The reconstruction result obtained using an optimal choice of minimum required measurements matches very closely (within 7%) with the one obtained using all measurements. The higher recovery of μ_a value in the tumor region was due to the assumption of uniform μ_s' across regions. Note that the heterogeneous target μ_s' in this case has no effect on the choice of optimal M.

In summary, we have presented a novel approach based on data-resolution characteristics of the imaging problem for optimally choosing the minimum required measurements for performing image-guided diffuse optical tomography. We also showed that such a choice yields reconstruction results that are similar to the ones obtained using all measurements.

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