DS286 | 2016-09-09

L9: Queues

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Slides courtesy Venkatesh Babu, CDS

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Queue ADT

- FIFO Principle
- Elements **inserted only at rear** (enqueued) end and **removed from front** (dequeued)
  - Also called “Head” and “Tail”

Queue -Methods

- queue **New**() – Creates and returns an empty queue
- **Enqueue**(item v) – Inserts object **v** at the *rear* of the queue
- item **Dequeue**() – Removes the object from *front* of the queue. Error occurs if the queue is empty
- item **Front**() – Returns, but does not remove the front element. An error occurs if the queue is empty

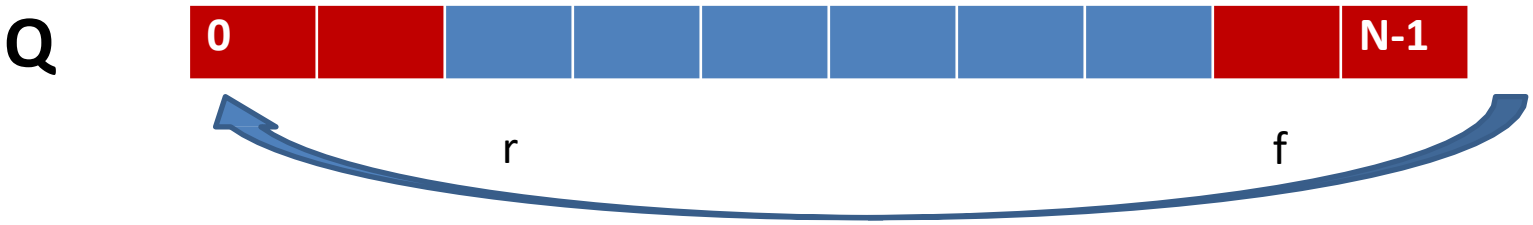
Queue – Methods & Invariants

- int **Size**() – number of items in queue
- boolean **IsEmpty**() – is size == 0
- Axioms/invariants
  - `Front(Enqueue(New(),v)) = v`
  - `Dequeue(Enqueue(New(), v)) = New()`
  - `Front(Enqueue(Enqueue(Q, w), v)) = Front(Enqueue(Q, w))`
  - `Dequeue(Enqueue(Enqueue(Q, w), v))= Enqueue(Dequeue(Enqueue(Q, w)), v)`

Array Implementation of Queue

- Using array in *circular* fashion
  - Wraparound using mapping function (recollect from List ADT discussion)
- A max size N is specified
- Q consists of an N element array and 2 integer variables having array index:
  - **f**: index of the front element (head, for dequeue)
  - **r**: index of the element after the rear one (tail, for enqueue)

Array Implementation of Queue



- What does f=r mean ?

- Resolve Ambiguity:
  - We will never add n^{th} element to Queue (declare full if the size of queue is N-1) .

Pseudo Code

- **int size()**

```
Return (N-f+r) mod N
```

- **bool isEmpty()**

```
Return(f==r)
```

- **int front()**

```
If isEmpty() then Return QueueEmptyException
Else Return Q[f]
```

Pseudo Code

- **int Dequeue()**

```
If isEmpty() then Return QueueEmptyException
v = Q[f]
Q[f] = null
f = (f+1) mod N
Return v
```

- **Enqueue(v)**

```
If size()==n-1 then Return QueueFullException
Q[r] = v
r = (r+1) mod N
```

Compute Complexity? Storage Complexity?

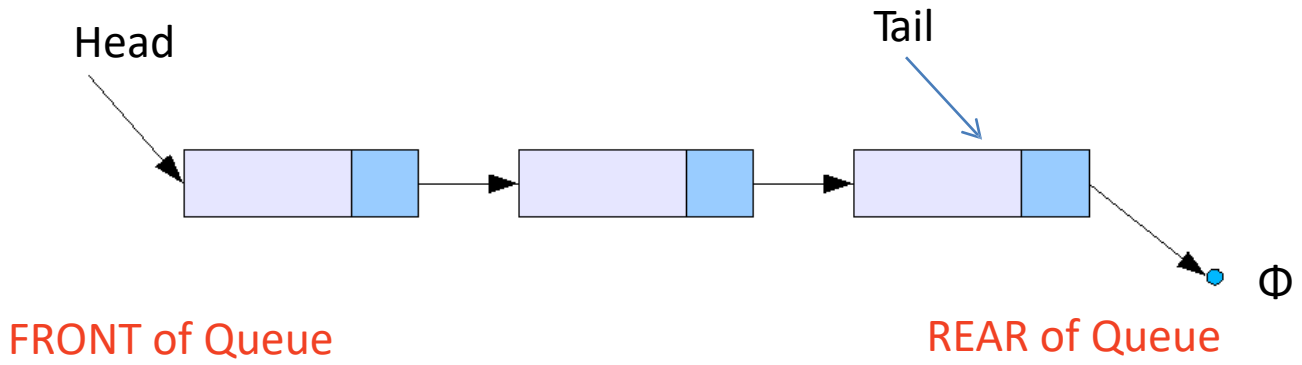
Linked List

- Problem with array: Requires the number of elements *a priori*.

Implementation with linked List

NOTE: Different from what was mentioned in class.

Nodes (data, pointer) connected in a chain by links



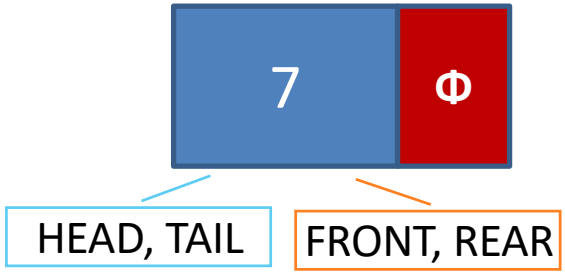
- **Maintain two pointers, to head and tail of linked list.**
- **The head of the list is FRONT of the queue, the tail of the list is REAR of the queue.**
- **Why not the opposite?**

Example

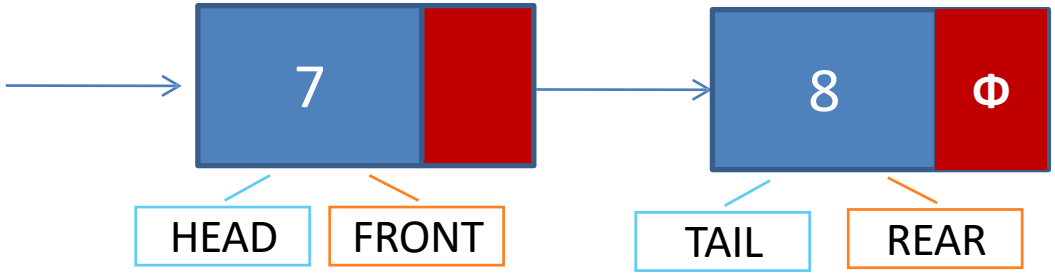
Tail of linked list is REAR of queue… Enqueue at tail
Head is FRONT of queue… Dequeue at head

TAIL = Φ, HEAD = Φ

- Enqueue 7



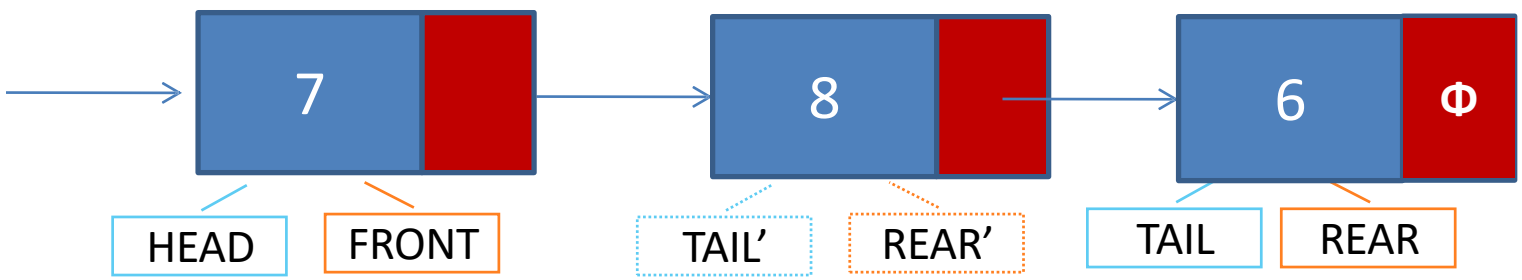
- Enqueue 8



Example

Tail of linked list is REAR of queue… Enqueue at tail
Head is FRONT of queue… Dequeue at head

- Enqueue 6



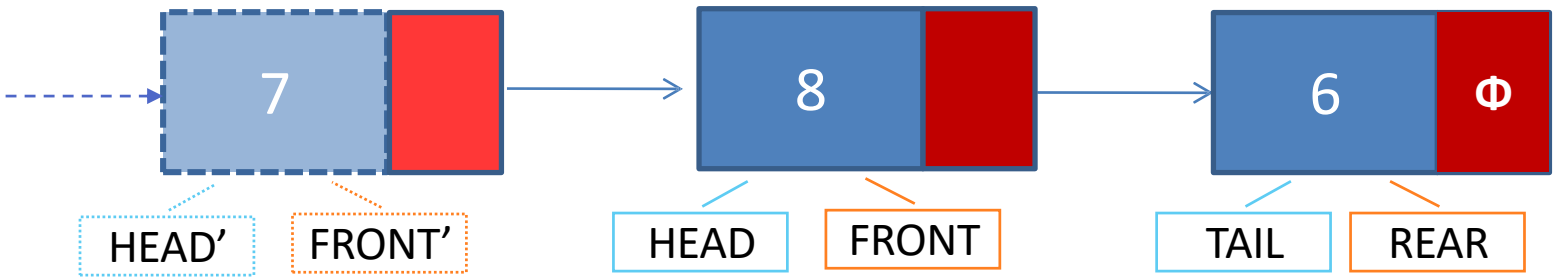

```
Node n = new Node(6)
n.next = null
TAIL.next = n
TAIL = n          // NEW REAR
```

O(1) complexity to enqueue

Example

Tail of linked list is REAR of queue… Enqueue at tail
Head is FRONT of queue… Dequeue at head

- Dequeue → 7

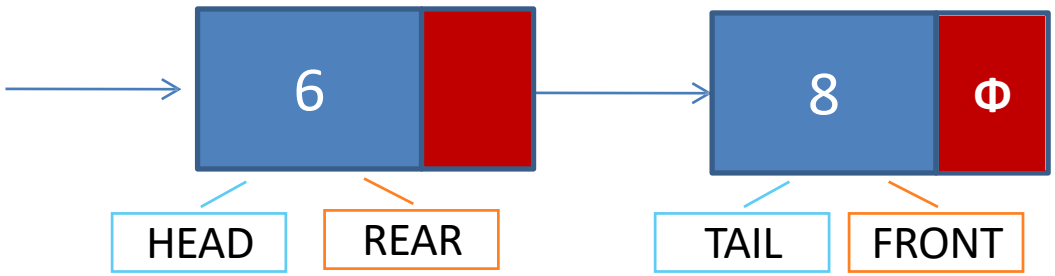



```
int v = HEAD.value
tmp = HEAD.next
delete(HEAD)
HEAD = tmp      // NEW HEAD
return v            // 7
```

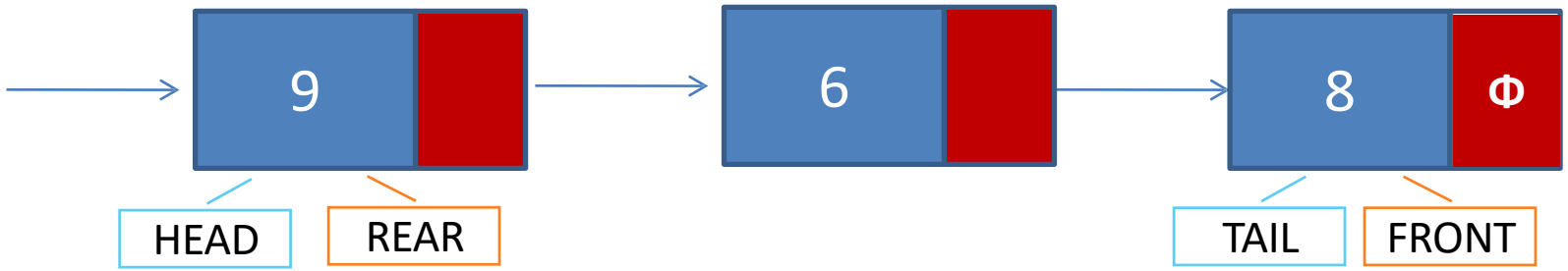
O(1) complexity to dequeue

Example

What if Head of linked list is REAR of queue, Tail the FRONT?



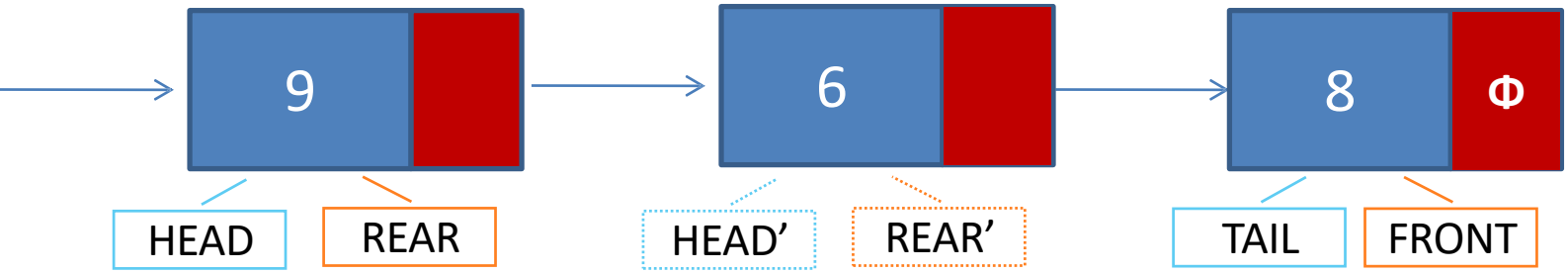
- Enqueue 9



Example

What if Head of linked list is REAR of queue, Tail the FRONT?

- Enqueue 9



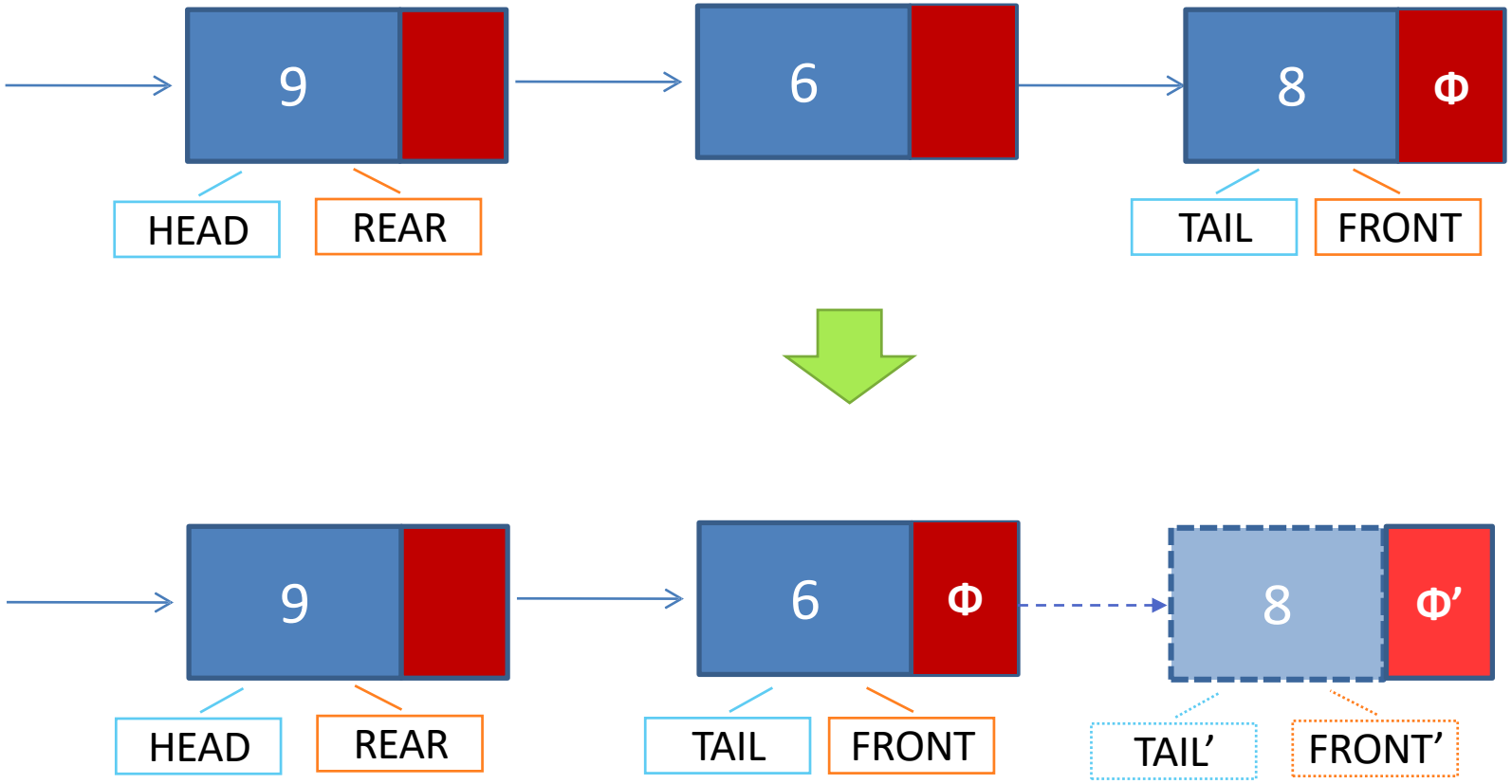

```
Node n = new Node(9)
n.next = HEAD
HEAD = n
```

O(1) complexity to enqueue

Example

What if Head of linked list is REAR of queue, Tail the FRONT?

- Dequeue → 8

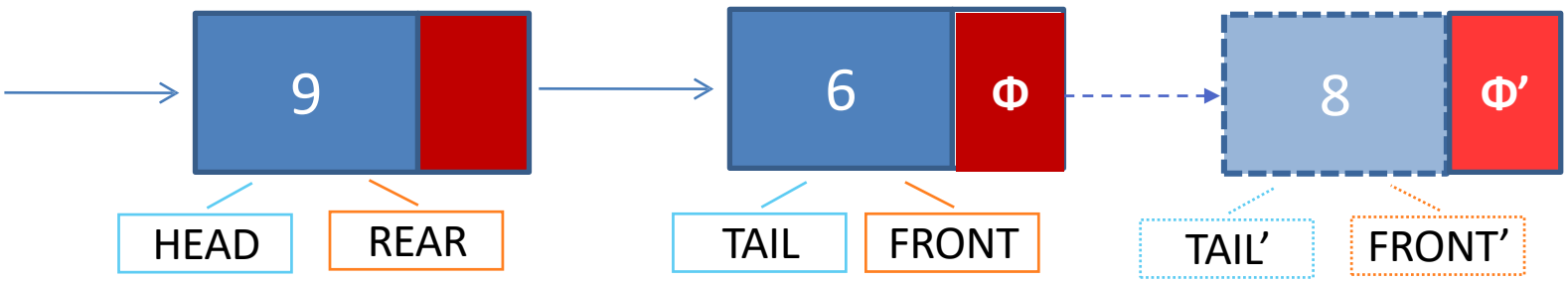


Example

- Dequeue → 8

What if Head of linked list is REAR of queue, Tail the FRONT?

Things don't work well!




```
int v = TAIL.value              // 8
// tail's "previous" (6) should point to null!
n = HEAD                  // Can head be null?
while(n.next != TAIL) n = n.next;
n.next = null
delete(TAIL)
TAIL = n
return v
```

O(N) complexity to dequeue

Linked List


```
typedef struct Node {
  int value;
  struct Node *next;    // pointer to Node
} a_node;
```

Any linked list is a pointer to a node

```
typedef Node *list;     // head of list
```

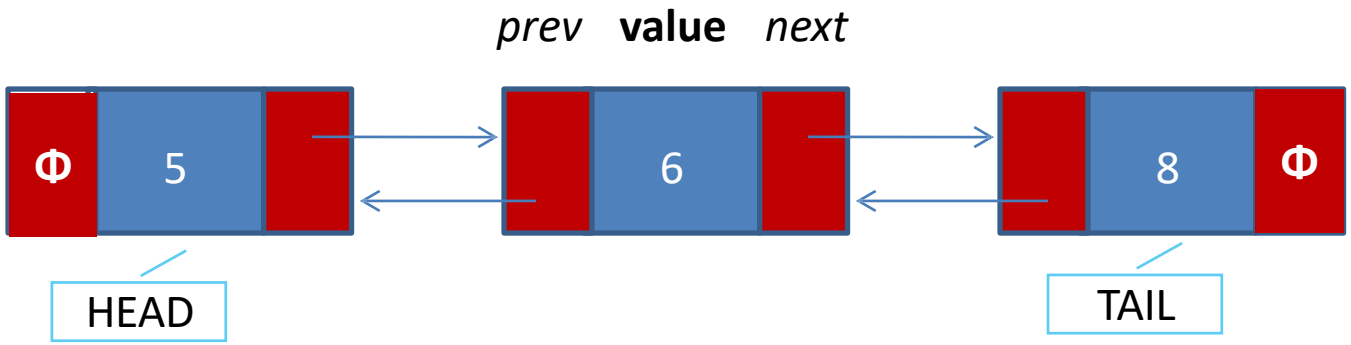
Double-Ended Queue (Dequeue)

- Supports insertion & deletion from front & rear
- **Supports six methods**
  - InsertFirst(item o) – Inserts 'o' at the beginning of deque
  - InsertLast(item o) – Inserts 'o' at the end of deque
  - item RemoveFirst() – removes the 1st element
  - item RemoveLast() – removes the last element
  - item First() – return first element
  - item Last() - return last element

Problem in implementing using single linked list with O(1)

e.g. If head is front and tail is rear, RemoveLast will require traversal from head to tail's previous

Dequeue as Doubly Linked List

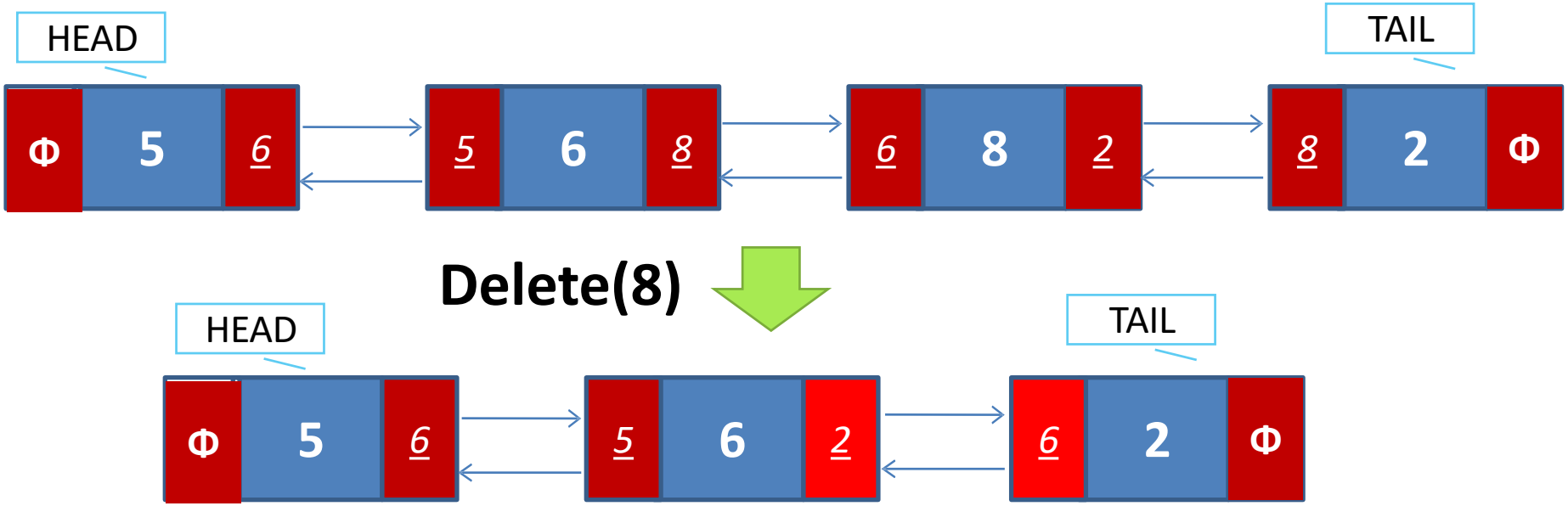


- Nodes of doubly linked list have a next and a prev link
- All the methods of a dequeue using doubly linked list have a constant running time O(1)
  - *How?*

Doubly Linked List


```
typedef struct Node {
  int value;
  struct Node *next, *prev;
} a_node;
```

Delete Element from Doubly Linked List




```
Node n = HEAD
while(n.next.value != v) n = n.next;
tmp = n.next              // n = 6, tmp = 8
n.next = tmp.next         // 6.next = 2
tmp.next.prev = n         // 2.prev = 6
delete(tmp)               // delete 8
```

Implement Stacks with Deques

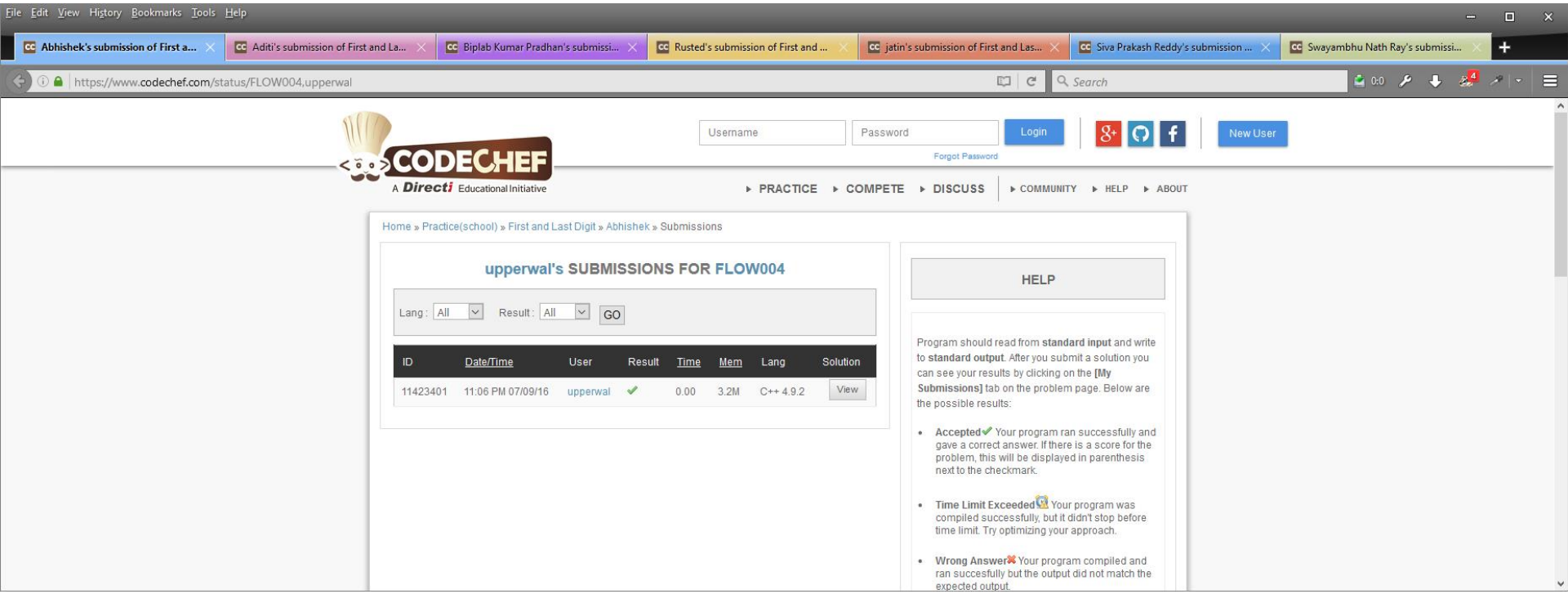
Stack Method	Deque Method
Size()	Size()
IsEmpty()	IsEmpty()
Top()	Last()
Push()	insertLast()
Pop()	removeLast()

Implement Queue with Deques

Stack Method	Deque Method
Size()	Size()
IsEmpty()	IsEmpty()
front()	first()
enqueue()	insertLast()
dequeue()	removeFirst()

CodeChef: FLOW004

- 9/Sep



Tasks

- Solve sanity check problem on CodeChef by **Sep 14**
  - **`https://www.codechef.com/problems/FLOW004`**
- Self study (Sahni Textbook)
  - **Check**: Have you read Chapter 8 “Stacks”? Solved exercises?
  - **Read**: Chapter 9, Queues from textbook
  - **Try:** Exercise 4, 14, 18 from Chapter 9 of textbook

Questions?

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